INFORMATIONAL ESTIMATES OF A TRANSLOG COST FUNCTION†

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Abstract

This study compares the estimation of share equations derived from the Translog cost function under maximum likelihood and minimum information. The sample RMSEs are consistently lower under maximum likelihood.

keywords: maximum likelihood, minimum information, cost shares

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1. Introduction

Dual objective functions and flexible functional forms, when used in conjunction with well known envelope theorems (e.g. Shephard's and Hotelling's lemmas) provide a theoretically rigorous means of specifying systems of demand (and supply) functions with a minimum of a priori restrictions on the assumed technology sets or preference orderings. While the advantages of this "approach" are many, there are limitations as well. One significant estimation problem relates to parsimony; unrestricted flexible functions characterizing N factors (inputs or consumption goods) require (N+1)(N+2)/2 parameters. Additionally, asymptotically efficient estimators such as maximum likelihood (ML) or iterated SUR require estimation of an additional N(N+1)/2 covariance elements. For example, if four inputs (goods) are included (e.g. capital, labor, energy and intermediate goods as in Diewert and Wales, 1995) 15 parameters and 10 covariance terms must be estimated. Accordingly, for many post-war aggregate time series data sets, estimation of an unrestricted flexible function and the associated covariances calls into question the validity of the asymptotic properties associated with maximum likelihood.

An alternative to using ML estimation is the informational estimator proposed by Finke, Flood and Theil (1984); Finke and Theil (1984); and Theil and Fiebig (1984). Informational estimation involves minimizing the Strobel (1992) deviation between actual budget shares and the predictions obtained from the estimated share equations. Since estimates of the covariance matrix are not required, informational estimators provide an attractive alternative to ML in small sample situations. Theil and Chen (1995) demonstrated that the informational estimator performed as well as ML with known covariance matrix and better than ML with an unknown covariance matrix in terms of root mean square errors (RMSE).[1]

Perhaps the most prevalent functional form used in empirical analysis is the Translog cost (expenditure function). Since estimation of the share equations obtained from the Translog often occurs in small samples and additionally requires the estimated covariance matrix, it seems likely that the efficiency gains demonstrated by Theil and Chen may also be realized for share equations obtained from the Translog. The purpose of this paper is to investigate any efficiency gains obtained from informational estimators of Translog share equations when compared to ML estimator. Additionally, the methodology employed by Theil and Chen is extended by using bootstrapping techniques rather than simulating model coefficients according to a given distribution.

2. Model Specification and Informational Estimation

The general form of the translog cost function for n-variable inputs and m-outputs can be represented

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[1] For a 10 good system with 15 countries, ML with unknown covariance failed to converge while again the estimates of the informational estimator were compared with ML with known covariances.

by:

$$\begin{gathered} ln(C) = \alpha_0 + \sum_{i=1}^{n} \alpha_i ln(p_i) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} ln(p_i) ln(p_j) + \sum_{i=1}^{m} \beta_i ln(q_i) + \\ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} B_{ij} ln(q_i) ln(q_j) + \sum_{i=1}^{n} \sum_{j=1}^{m} \Gamma_{ij} ln(p_i) ln(q_j) \\ \sum_{i=1}^{n} \alpha_i = 1 \\ \sum_{i=1}^{n} A_{ij} = 0 \;\; j = 1, ...n \end{gathered} \tag{1}$$

where $ln(C)$ is the (natural) logarithm of total cost, $ln(p_i)$ is the logarithm of the input price for input i, $ln(q_i)$ is the logarithm of the output quantity i, and α_i, β_i, A_{ij}, B_{ij} and Γ_{ij} are estimated parameters. Taking the logarithmic derivatives of equation (1) with respect to input prices and invoking Shephard's lemma yields a system of input share equations:

$$w_i = \alpha_i + \sum_{j=1}^{n} A_{ij} ln(p_j) + \sum_{j=1}^{m} \Gamma_{ij} ln(q_j) \;\; i = 1, ...n \tag{2}$$

where w_i denotes the cost share of the i*th* factor of production.

Informational estimation minimizes the information of the posterior sample:

$$\min_{\alpha, A, \Gamma} \sum_{t=1}^{T} \left[\sum_{i=1}^{n} w_{tj} ln \left(\frac{w_{ti}}{\tilde{w}_{ti}(\theta)} \right) \right] \tag{3}$$

where w_{ti} is the observed budget share of input i for the t*th* observation, $\tilde{w}_{ti}(\theta)$ is the estimated budget share for the i*th* input for the t*th* observation, and θ denotes the parameters associated with equation (2). Note, that if the estimated cost shares exactly equals the observed cost shares, the informational value is zero. To operationalize the estimation process, the theoretical requirement that the cost shares sum to one must be imposed and the expression

$$\max_{\alpha, A, \Gamma} \sum_{t=1}^{T} \left[\sum_{i=1}^{n-1} w_{ti} ln(\tilde{w}_{ti}(\theta)) + w_{tn} ln \left(1 - \sum_{i=1}^{n-1} \tilde{w}_{ti}(\theta) \right) \right] \tag{4}$$

must be solved with the further restriction that $0 < \tilde{w}_{ti}(\theta) < 1$ for all t,j. Equation (3) is estimated with Minos 5.1 (Murtauh and Saunders) subject to

$$\delta \leq \tilde{w}_{ti}(\theta) = \alpha_i + \sum_{j=1}^{n} A_{ij} ln(p_{tj}) + \sum_{j=1}^{m} \Gamma_{ij} ln(q_{tj}) \leq 1 - \delta$$

for all t,j where $ln(p_{tj})$ and $ln(q_{tj})$ denote the observed log of input prices and output quantities, respectively.[2]

3. Sample Based Standard Errors

To examine the properties of informational estimation, the share equations for commerical banks in the state of Florida were estimated. Inputs included debt capital, wages and fixed premises. Outputs included real estate loans, agricultural loans, other loans, securities and transaction deposits. The sample was limited to the 277 banks without foriegn deposits during 1989. The maximum likelihood (ML) and minimum informational (MI) estimates are presented in Table 1. The results of ML typically agree in sign and magnitude with those of MI with the exception of α_2 which changes in sign.

[2] The general shape of the information function in equation (3) suggests that this condition will hold at optimum. However, if the function is numerically optimized, a particular iteration may fail to meet this condition yielding a failure in the search for the maximum. Accordingly the solution is checked at optimum to guarantee that none of these constraints are binding. If a constraint is binding δ is set to a smaller value and the problem resolved.

In theory, the covariance matrix for the maximum likelihood estimates can be estimated using the Hessian of the likelihood function at optimum. However, Laitinen (1979) and Meisner (1982) call into question the use of tests based on this matrix. Laitinen shows that the standard test for homogeniety rejects the null hypothesis too frequently. Meisner obtains a similar conclusion for tests of symmetry. Thus, the standard errors for both ML and MI are obtained using bootstrapping.

The results in Table 1 indicate that ML always yields a smaller root mean squared error (RMSE) than MI in full sample.[3] Thus, we conclude that maximum likelihood is preferred in large samples. Next to test the effect of sample size on the RMSE, we reduce the sample size to 200, 100, 50, and 25 observations. These results, also presented in Table 1, indicate that ML yields a consistently smaller RMSE at each sample level. This result would appear inconsistent with the results of Theil and Chen.

Dividing the RMSE for each sample size by the RMSEs for the full sample indicates that the relative increase in RMSE is smaller for MI than ML. This result would appear consistent with the results of Theil and Chen, but an extremely small sample may be required for MI to yield smaller RMSEs than ML.

4. Conclusions

This study examines the use of minimum information estimation in the analysis of Translog cost functions. The empirical analysis indicates that the root mean square errors of the maximum likelihood estimator are always smaller than the root mean square errors of the minimum information estimator. These results appear to contradict the results of Theil and Chen who find that the minimum information approach dominates maximum likelihood yielding smaller root mean square errors in small samples. The differences in the results may be explained partially by experimental design. Theil and Chen start by assuming a true model and simulate samples assuming multivariate normality while this study uses the bootstrapping procedure without *a priori* restrictions on the error structure. One artifact of the bootstrapping procedure is that the bootstrapped means from the minimum information approach differs from the original sample mean. The driving force behind this divergence is the lack of a zero–residual condition in the minimum information estimator, unlike the maximum likelihood estimator. Thus, the sum of errors under minimum information for the full sample is .297 for the first equation and -.428 for the second equation. These results are compared with zero for both equations under maximum likelihood. In addition to calling into question the relative power of minimum information, the difference between the bootstrapped means and the original estimates may also raise questions about the consistency of the minimum information approach.

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[3] The RMSEs are based on 25,000 draws from the estimated residuals and the orginal price and quantity data. The "true" parameters for the ML experiments are those obtained from the original ML estimation for each sample size while those for the MI experiments are obtained from the original MI estimation of each sample

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	α_1	α_2	A_{11}	A_{12}	A_{22}	Γ_{11}	Γ_{21}	Γ_{31}	Γ_{41}	Γ_{51}	Γ_{12}	Γ_{22}	Γ_{32}	Γ_{42}	Γ_{52}
Full Sample															
Maximum Likelihood															
Original Estimate	0.706	0.063	0.136	-0.099	0.087	0.049	-0.001	0.000	0.025	-0.008	-0.037	0.001	0.000	-0.010	-0.009
Bootstrapped Mean	0.707	0.063	0.136	-0.099	0.087	0.049	-0.001	0.000	0.025	-0.008	-0.037	0.001	0.000	-0.010	-0.009
RMSE	0.103	0.093	0.014	0.012	0.011	0.007	0.003	0.000	0.006	0.008	0.005	0.003	0.000	0.004	0.006
Minimum Information															
Original Estimate	0.875	-0.140	0.151	-0.122	0.115	0.050	-0.001	0.000	0.024	-0.013	-0.036	0.000	0.000	-0.010	0.013
Bootstrapped Mean	1.142	-0.152	0.144	-0.144	0.144	0.049	0.001	0.000	0.016	-0.018	-0.049	-0.001	0.000	-0.016	0.018
RMSE	0.290	0.113	0.016	0.026	0.032	0.006	0.003	0.000	0.010	0.009	0.014	0.003	0.000	0.008	0.009
First 200 Observations															
Maximum Likelihood															
Original Estimate	0.606	0.162	0.130	-0.092	0.079	0.048	0.001	0.000	0.039	-0.016	-0.039	0.000	0.000	-0.017	0.012
Bootstrapped Mean	0.607	0.161	0.130	-0.092	0.079	0.048	0.001	0.000	0.039	-0.016	-0.039	0.000	0.000	-0.016	0.012
RMSE	0.126	0.115	0.017	0.015	0.014	0.008	0.004	0.000	0.007	0.009	0.007	0.003	0.000	0.006	0.007
Minimum Information															
Original Estimate	0.764	-0.036	0.142	-0.114	0.107	0.048	0.002	0.000	0.038	-0.021	-0.039	0.000	0.000	-0.016	0.016
Bootstrapped Mean	1.005	-0.015	0.132	-0.132	0.132	0.048	0.002	0.000	0.026	-0.022	-0.048	-0.002	0.000	-0.026	0.022
RMSE	0.276	0.138	0.020	0.024	0.030	0.008	0.004	0.000	0.014	0.009	0.012	0.004	0.000	0.012	0.011
First 100 Observations															
Maximum Likelihood															
Original Estimate	0.624	0.073	0.098	-0.087	0.088	0.052	0.004	0.000	0.064	-0.059	-0.041	-0.002	0.000	-0.036	0.041
Bootstrapped Mean	0.627	0.073	0.097	-0.086	0.088	0.051	0.004	0.000	0.064	-0.059	-0.041	-0.002	0.000	-0.035	0.041
RMSE	0.173	0.149	0.023	0.019	0.018	0.012	0.005	0.000	0.010	0.012	0.009	0.004	0.000	0.008	0.009
Minimum Information															
Original Estimate	0.783	-0.105	0.111	-0.108	0.112	0.050	0.004	0.000	0.062	-0.060	-0.040	-0.002	0.000	-0.034	0.044
Bootstrapped Mean	1.045	-0.055	0.123	-0.123	0.123	0.048	0.003	0.000	0.044	-0.051	-0.048	-0.003	0.000	-0.044	0.051
RMSE	0.321	0.193	0.026	0.027	0.025	0.011	0.005	0.000	0.021	0.014	0.014	0.004	0.000	0.013	0.013
First 50 Observations															
Maximum Likelihood															
Original Estimate	0.557	0.122	0.074	-0.070	0.075	0.038	0.010	0.000	0.064	-0.053	-0.028	-0.008	0.000	-0.036	0.036
Bootstrapped Mean	0.557	0.123	0.074	-0.070	0.075	0.038	0.010	0.000	0.064	-0.053	-0.027	-0.008	0.000	-0.036	0.036
RMSE	0.225	0.203	0.030	0.026	0.025	0.018	0.007	0.000	0.014	0.015	0.014	0.005	0.000	0.011	0.012
Minimum Information															
Original Estimate	0.639	0.112	0.080	-0.082	0.089	0.038	0.010	0.000	0.065	-0.057	-0.027	-0.008	0.000	-0.037	0.040
Bootstrapped Mean	0.900	0.090	0.095	-0.095	0.095	0.034	0.010	0.000	0.049	-0.049	-0.034	-0.010	0.000	-0.049	0.049
RMSE	0.350	0.245	0.032	0.032	0.029	0.017	0.006	0.000	0.020	0.015	0.018	0.006	0.000	0.018	0.016
First 25 Observations															
Maximum Likelihood															
Original Estimate	0.078	0.407	0.042	-0.053	0.066	0.087	0.009	0.000	0.091	-0.095	-0.073	-0.005	0.000	-0.050	0.073
Bootstrapped Mean	0.083	0.404	0.043	-0.053	0.066	0.087	0.009	0.000	0.091	-0.095	-0.073	-0.005	0.000	-0.050	0.073
RMSE	0.256	0.249	0.031	0.029	0.030	0.026	0.008	0.001	0.019	0.021	0.020	0.006	0.001	0.015	0.017
Minimum Information															
Original Estimate	0.212	0.284	0.052	-0.067	0.078	0.083	0.008	0.000	0.086	-0.091	-0.073	-0.004	0.000	-0.045	0.073
Bootstrapped Mean	0.518	-0.472	0.074	-0.074	0.074	0.087	0.004	0.000	0.058	-0.082	-0.087	-0.004	0.000	-0.058	0.082
RMSE	0.411	0.332	0.039	0.033	0.033	0.023	0.008	0.001	0.033	0.021	0.027	0.007	0.001	0.021	0.021