

Factor Bias, Technical Change, and Valuing Research: Lecture XXVII

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1 Mathematical Model of Technical Change

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- TFP Growth in the Southeast
- TFP Growth Versus R & D Stock
- Cointegration Estimator

Mathematical Model of Technical Change

- Moss, C.B., T.G. Schmitz, and A. Schmitz. 2005. The New Institutional Economics, Schumpeter, and E-Commerce in *E-Commerce in Agribusiness* eds. T.G. Schmitz, C.B. Moss, A. Schmitz, A. Kagan, and B. Babcock, Florida Science Source, Inc., 3-24.
- If we start from the quadratic production function as

$$f(x_1, x_2) = a_0 + a_1x_1 + a_2x_2 + \frac{1}{2} (A_{11}x_1^2 + 2A_{12}x_1x_2 + A_{22}x_2^2) \quad (1)$$

- Assuming an output price of p and input prices of w_1 and w_2 for inputs x_1 and x_2 , respectively, the derived demands for each input can be expressed as

$$\begin{aligned} x_1^*(p, w_1, w_2) &= \frac{A_{12}a_2p - A_{22}a_1p + A_{22}w_1 - A_{12}w_2}{p(A_{11}A_{22} - A_{12}^2)} \\ x_2^*(p, w_1, w_2) &= \frac{A_{12}a_1p - A_{11}a_2p - A_{12}w_1 + A_{11}w_2}{p(A_{11}A_{22} - A_{12}^2)} \end{aligned} \quad (2)$$

- In order to analyze the possible effect of technological change, we hypothesize an input augmenting technical change similar to the general form of technological innovation introduced by Hayami and Ruttan.

- We introduce two functions

$$\begin{aligned}x_1^* &= \gamma_1(\psi) x_1 \\ x_2^* &= \gamma_2(\psi) x_2\end{aligned}\tag{3}$$

where $\gamma_1(\psi)$ and $\gamma_2(\psi)$ are augmentation factors and ψ is a technological change.

- Hence, $\gamma_1(\psi), \gamma_2(\psi) \geq 1$ for any $\psi > 0$.

- Thus, technological change increases the output created by each unit of input. Integrating these increases into the forgoing production framework, the derived demands for each input becomes

$$x_1^*(p, w_1, w_2, \psi) =$$

$$\frac{A_{12}a_2\gamma_1(\psi)\gamma_2(\psi)p - A_{22}a_1\gamma_1(\psi)\gamma_2(\psi)p + A_{22}\gamma_2(\psi)w_1 - A_{12}\gamma_1(\psi)w_2}{p(A_{11}A_{22} - A_{12}^2)\gamma_1^2(\psi)\gamma_2(\psi)}$$

$$x_2^*(p, w_1, w_2, \psi) =$$

$$\frac{A_{12}a_1\gamma_1(\psi)\gamma_2(\psi)p - A_{11}a_2\gamma_2(\psi)\gamma_2(\psi)p - A_{12}\gamma_2(\psi)w_1 + A_{22}\gamma_1(\psi)w_2}{p(A_{11}A_{22} - A_{12}^2)\gamma_1(\psi)\gamma_2^2(\psi)}$$

(4)

- In order to simplify our discussion, we assume that the new technology does not affect the effectiveness of x_2 , or $\gamma_2(\psi) \rightarrow 1$.
 - Under this assumption the derived demand for each input becomes

$$x_1^*(p, w_1, w_2, \psi) = \frac{A_{12}a_2\gamma_1(\psi)pA_{22}a_1\gamma_1(\psi)p + A_{22}w_1 - A_{12}\gamma_1(\psi)w_2}{p(A_{11}A_{22} - A_{12}^2)\gamma_1^2(\psi)}$$

$$x_2^*(p, w_1, w_2, \psi) = \frac{A_{12}a_1\gamma_1(\psi) - A_{11}a_2\gamma_1(\psi)p - A_{12}w_1 + A_{22}\gamma_1(\psi)w_2}{p(A_{11}A_{22} - A_{12}^2)\gamma_1(\psi)}$$

(5)

- Finally, we tack the derivative of each derived demand curve with $\gamma_1(\psi) \rightarrow 1$ (or the instantaneous change in technology) yielding

$$\begin{aligned} \left. \frac{\partial x_1^*(p, w_1, w_2, \psi)}{\partial \psi} \right|_{\gamma_1(\psi) \rightarrow 1} &= \frac{2(A_{12}a_2p - A_{22}a_1p + A_{22}w_1 - A_{12}w_2)\gamma_1'(\psi)}{(A_{11} - A_{12}^2)p} \\ &\quad + \frac{(A_{12}a_2p - A_{22}a_1p - A_{12}w_2)\gamma_1'(\psi)}{(A_{11}A_{22} - A_{12}^2)p} \\ \left. \frac{\partial x_2^*(p, w_1, w_2, \psi)}{\partial \psi} \right|_{\gamma_1(\psi) \rightarrow 1} &= -\frac{(A_{12}a_1p - A_{11}a_2p - A_{12}w_1 + A_{11}w_2)\gamma_1'(\psi)}{(A_{11}A_{22} - A_{12}^2)p} \\ &\quad + \frac{(A_{12}a_1p - A_{11}a_2p + A_{11}w_2)\gamma_1'(\psi)}{(A_{11}A_{22} - A_{12}^2)p} \end{aligned}$$

(6)

Valuing State Level Research

- Moss, C.B. 2006. Valuing State Level Funding for Research: Results for Florida. *Journal of Agricultural and Applied Economics* 38(1): 169–183.
- The most basic definition of productivity involves the quantity of output that can be derived from a fixed quantity of inputs.
- For example, most would agree that a gain in productivity has occurred if corn yields increased from 70 bushels per acre to 75 bushels per acre given the same set of input (i.e., pounds of fertilizer, or hours of labor).
- Aggregate agricultural outputs and inputs could be computed based on Divisia quantity indices.
 - Specifically, Y_t let be the aggregate output index computed as

$$Y_t = \sum_{i=1}^n r_{it} y_{it} \quad (7)$$

where r_{it} is the revenue share of output i .

- Continued

- Similarly, the aggregate input index can be computed as X_t

$$X_t = \sum_{j=1}^m s_{jt} x_{jt} \quad (8)$$

where s_{jt} is the cost share of input j .

- Equating aggregate output with aggregate input yields

$$Y_t = \gamma_t X_t \quad (9)$$

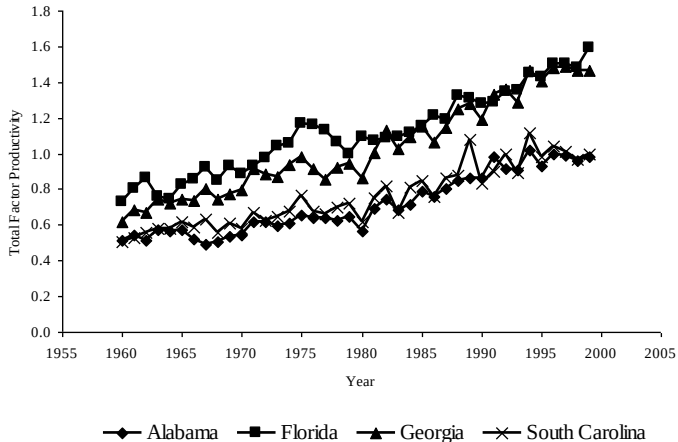
- Rearranging slightly yields

$$\frac{Y_t}{X_t} = \gamma_t \quad (10)$$

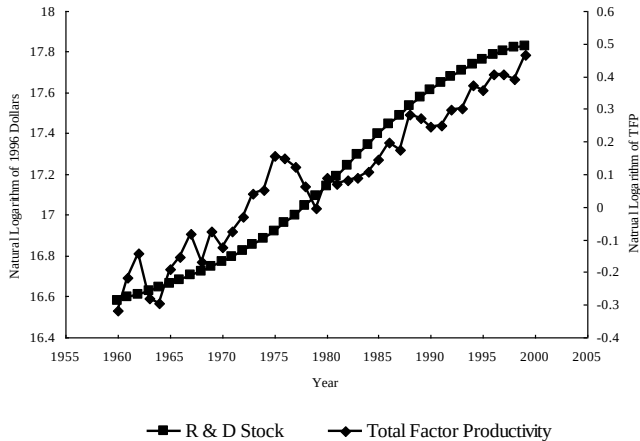
- The rate of technical change can be derived from the log change in both sides

$$\ln \left(\frac{Y_t}{X_t} \right) - \ln \left(\frac{Y_{t-1}}{X_{t-1}} \right) = \ln \left(\frac{\gamma_t}{\gamma_{t-1}} \right) \quad (11)$$

TFP Growth in the Southeast



TFP Growth Versus R & D Stock



Cointegration Estimator

- The Johansen (1988) approach involves estimating a vector error-correction mechanism expressed as

$$\Delta x_t = \Pi x_{t-1} + \sum_{i=1}^k \Gamma_i \Delta x_{t-i} + \Phi D_t + \epsilon_t \quad (12)$$

where x_t is a vector of endogenous variables, Δx_t denotes the time-difference of that vector, D_t is a vector of exogenous variables, ϵ_t is a vector of residuals, and Π , Γ_i , and Φ are estimated parameters.

- If a long-run relationship (e.g., cointegrating vector) exists, the Π matrix is singular ($\Pi = \alpha'\beta$).
 - The β vector is the cointegrating vector or long-run equilibrium.

- Continued

- The statistical properties of the cointegrating vector are determined by the eigenvalues of the estimated Π matrix.
 - Denoting λ_i represent the i^{th} eigenvalue (in descending order of significance), the test for significance of the cointegrating vector can be written as

$$2 \ln [Q (H_1 (r) | H_1 (p))] = -T \sum_{i=r+1}^p \ln (1 - \hat{\lambda}_i) \quad (13)$$

which tests the hypothesis that r cointegrating vectors are present, $H_1 (r)$, against the hypothesis that p cointegrating vectors are present, $H_1 (p)$.

- The existence of a cointegrating vector in this framework implies that the linear combination (z_t) of the natural logarithm of TFP and research and the natural logarithm of research and development stocks (RD_t) is stationary, or a long-run equilibrium between these two series exists.
 - While this cointegrating vector is not uniquely identified, the long-run relationship can be expressed as

$$z_t = \begin{bmatrix} 1.000 \\ -0.794 \end{bmatrix}' \begin{bmatrix} \ln(TFP_t) \\ \ln(RD_t) \end{bmatrix} + 13.433 \quad (14)$$

- Building on this expression, the long-run relationship can be expressed as

$$\ln(TFP_t) - 0.794 \ln(RD_t) + 13.433 = z_t \quad (15)$$

- Manipulating this result further yields

$$\frac{dTFP_t}{dRD_t} = 0.794 \frac{TFP_t}{RD_t} \quad (16)$$

- Continued
 - Using the geometric mean of both TFP and research and development stocks, TFP increases 0.0302 with a one million dollar increase in the research and development stock.
 - This number appears small, but it represents 113 percent of the average annual increase in productivity observed in the state.

- In order to understand the possible causes of the lack of a long-run equilibrium between agricultural profitability and productivity, I express the change in profit over time as

$$D(\pi_t) = D(F_t) + D(\Psi_t) \quad (17)$$

where π_t denotes profit in period t , F_t denotes Total Factor Productivity in time t , and Ψ_t denotes the change in relative price ratio in time t .

- In order to derive this relationship, we start with agricultural profit defined as

$$\pi_t = \sum_{i=1}^n p_{it} y_{it} - \sum_{j=1}^m w_{jt} x_{jt} \quad (18)$$

- Differentiating both sides yields

$$d\pi_t = \sum_{i=1}^n dp_{it} y_{it} + \sum_{i=1}^n p_{it} dy_{it} - \sum_{j=1}^m dw_{jt} x_{jt} - \sum_{j=1}^m x_{jt} dw_{jt} \quad (19)$$

- Continued

- Rewriting this expression using logarithmic differentiation yields

$$D(\pi_t) = \sum_{i=1}^n r_{it} D(p_{it}) + \sum_{i=1}^n r_{it} D(y_{it}) - \sum_{j=1}^m s_{jt} D(w_{jt}) - \sum_{j=1}^m s_{jt} D(x_{jt}) \quad (20)$$

- This expression can be rearranged to yield

$$D(\pi_t) = \left[\sum_{i=1}^n r_{it} D(y_{it}) - \sum_{j=1}^m s_{jt} D(x_{jt}) \right] + \left[\sum_{i=1}^n r_{it} D(p_{it}) - \sum_{j=1}^m s_{jt} D(w_{jt}) \right] \quad (21)$$