

# Measuring Changes in Productivity: XXV

Charles B. Moss<sup>1</sup>

<sup>1</sup>University of Florida

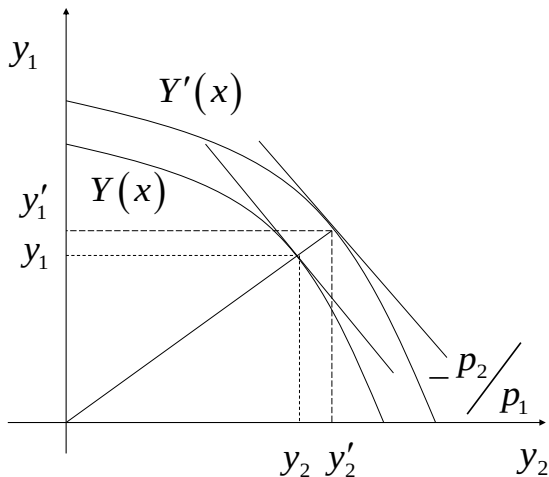
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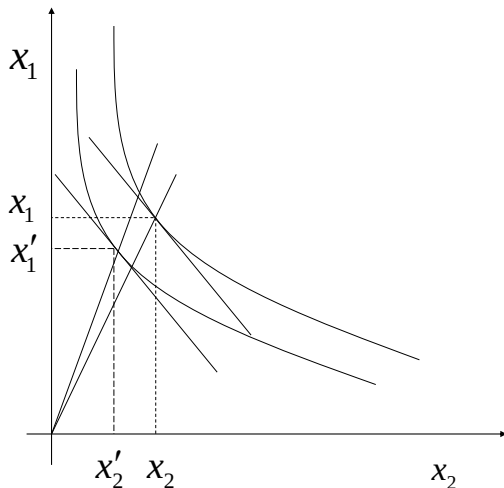
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# Defining Changes in Productivity



- In this figure, we assume that the inputs are constant at  $x$ , but the total level of outputs has increased from  $(y_1, y_2)$  to  $(y'_1, y'_2)$ .

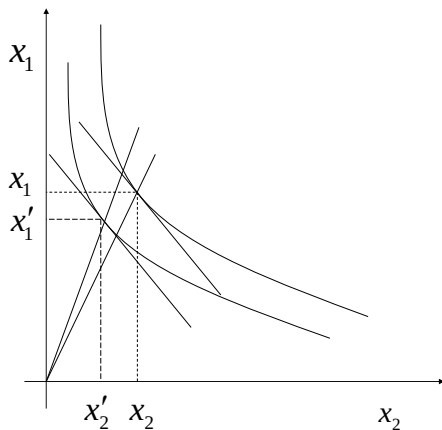
# Change in Productivity - Reduced Inputs



- In both cases, most would agree that a technical change has taken place.
- Further, most would agree that the technical change has increased the economic well being of society.
  - We now have more stuff for the same level of inputs.
- However, there are some issues that need to be raised.

- First, one could raise the question about embodied versus disembodied technical change.
  - This debate regards whether there has been an increase in knowledge, or whether there has been an increase in the quality of inputs.
  - If the increase in output has been associated with an increase in quality of an input, is it technical change?
  - For example, a large portion of the gains to research literature can be traced to Griliches discussion of hybrid corn.
  - In this case, the increase in technology was associated with the improvement in an input – seed.
  - More recently, some of the most recently observed increases in productivity may be traced to genetically modified organisms (GMOs).
  - Under most concepts of productivity, these increases do not represent changes in productivity in agriculture, but can be traced to changes in the input bundle.

- A second area of concern is whether the changes in technology are neutral with regard to the input bundle.
  - Going back to the figure, the increase in technology implies relatively more  $x_1$  it is biased toward  $x_1$ .



- Continued
  - In the hybrid corn example, additional fertilizer complemented the use of hybrid corn.
  - In the classic discussion, Hicks developed the notion of labor or capital augmenting technological development.

# Measuring Technological Change

- In the single input-single output analysis, one could directly measure technical change:

$$y = f(x, t) \Rightarrow \theta(t) = \frac{y}{x} \Rightarrow t = \theta^{-1}\left(\frac{y}{x}\right) \quad (1)$$

- Several factors should be considered.
  - We know that profit-maximizing behavior changes the point of production even in the univariate case.
  - We know that the decision maker chooses to produce where the marginal value product equals the price of the input.
    - Thus, if either the price of the input or the price of the output has changed, the ratio of outputs to inputs will change.
  - Even in the single variable case, we would wonder about excluded factors, things beyond the decision maker's control.

- Extending the analysis to the multivariate world, we begin by examining the product-product relationship in the first graph.
- Note

$$\theta(t) = \frac{Y'(x)}{Y(x)} = \frac{p_1 y'_1 + p_2 y'_2}{p_1 y_1 + p_2 y_2} \quad (2)$$

could be used as one measure of technical change.

- Similarly, in the input-input relationship:

$$\theta(t) = \frac{V'(y)}{V(y)} = \frac{w_1 x'_1 + w_2 x'_2}{w_1 x_1 + w_2 x_2} \quad (3)$$

- Note first that each of these formulations is based implicitly on Shephard duality:

$$\begin{aligned}\theta(t) &= \frac{Y'(x)}{Y(x)} = \frac{\pi(p, w, t = 1)}{\pi(p, w, t = 0)} \\ &= \frac{p_1 y'_1(p, w, t = 1) + p_2 y'_2(p, w, t = 1)}{p_2 y_1(p, w, t = 0) + p_2 y_2(p, w, t = 0)} \\ \theta(t) &= \frac{V'(y)}{V(y)} = \frac{c(w, y, t = 1)}{c(w, y, t = 0)} \\ &= \frac{w_1 x'_1(w, y, t = 1) + w_2 x'_2(w, y, t = 1)}{w_1 x_1(w, y, t = 0) + w_2 x_2(w, y, t = 0)}\end{aligned}\tag{4}$$

- More formally,

$$c(w, y, t) = \min_{x > 0} [w'x : x \in V(y, t)] \quad (5)$$

$$\Leftrightarrow V^*(y, t) = [x : w'x \geq c(w, y, t), w > 0]$$

- Thus, by gross simplification, we could envision a cost function

$$c(w, y, t) = \alpha_0 + \alpha'w + \frac{1}{2}w'Aw + \beta' + \frac{1}{2}y'By + w'\Gamma y + \theta(w, y, t) \quad (6)$$

$$x(w, y, t) = \alpha + AW + \Gamma y + \nabla_w \theta(w, y, t)$$

with  $\theta(w, y, t)$  being a measure of technical change.

- This formulation allows us to discuss several key features of technology measurement.
  - First, in the grossest sense, technological change tends to be a measurement of factors that we don't understand.
  - From the preceding equation, what is the difference between technology and a residual?
  - One approach is to proxy technical change with a simple time trend,  $t$ .
  - Alternatively, several studies have used other proxy variables such as spending on agricultural research.
- This formulation allows the researcher to examine the neutrality of technical change

$$\frac{\frac{\partial c(w, y, t)}{\partial w_i}}{\frac{\partial c(w, y, t)}{\partial w_j}} = \frac{x_i(w, y, t)}{x_j(w, y, t)} = \frac{x_i(w, y)}{x_j(w, y)} \quad (7)$$

- Finally, it is possible to envision adjusting this measure for differences in input quality.
  - For example, if the quality of one variable increases over time, then we could adjust the price of that variable upward to account for the increase in quality.

# Total Factor Productivity and Index Number Theory

- The index number approach can be looked at as an extension of the single input-single product scenario above

$$y = f(x, t) \Rightarrow \frac{dy}{dt} = \frac{\partial f(x, t)}{\partial x} \frac{dx}{dt} + \frac{\partial f(x, t)}{\partial t} \quad (8)$$

- Replacing differentiation with log differences

$$\begin{aligned} \frac{d \ln [y]}{dt} &= \sum_i \frac{\partial \ln [y]}{\partial \ln [x_i]} \frac{d \ln [x_i]}{dt} + T(x, t) \\ &= \sum_i \epsilon_i \frac{d \ln [x_i]}{dt} + T(x, t) \\ &= \sum_i \frac{w_i x_i}{p' y} \frac{d \ln [x_i]}{dt} + T(x, t) \\ \Rightarrow T(x, t) &= \frac{d \ln [y]}{dt} - \sum_i \frac{w_i x_i}{p' y} \frac{d \ln [x_i]}{dt} \end{aligned} \quad (9)$$

- This formulation is sometimes approximated as the Tornqvist-Theil measure

$$T(x, t) = \ln[y_t] - \ln[y_{t-1}] - \sum_{i=1}^N \frac{1}{2} [V_{it} + V_{i,t-1}] [\ln[x_{it}] - \ln[x_{i,t-1}]] \quad (10)$$

- Working backward

$$\begin{aligned} d \ln[y] &= d \ln[x] + T(x, t) \\ \Rightarrow d \ln[y] - d \ln[x] &= T(x, t) \\ \Rightarrow d \ln\left[\frac{y}{x}\right] &= T(x, t) \\ \Rightarrow d \ln\left[\frac{Q_y}{Q_x}\right] &= T(x, t) \end{aligned} \quad (11)$$

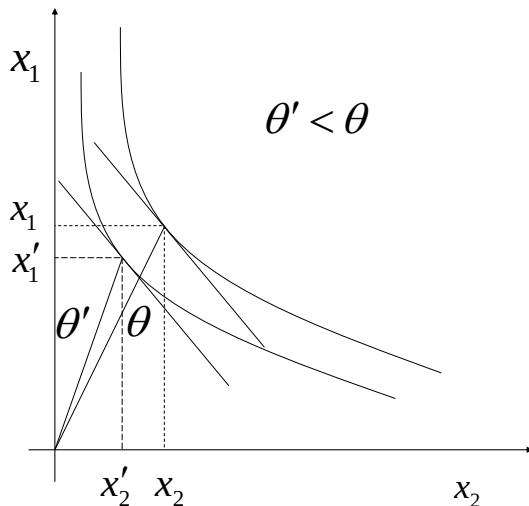
- In the Tornqvist-Theil index, the indices were implicitly Divisia output and input indices

$$Q_y = \sum_i \frac{p_i y_i}{\sum_j p_j y_j}, \quad Q_x = \sum_i \frac{w_i x_i}{\sum_j w_j x_j} \quad (12)$$

- The linkage in this case is the definition of total factor productivity

$$TFP = \frac{y}{x} \Rightarrow T\dot{F}P = \frac{\dot{y}}{\dot{x}} = \frac{\sum_i \frac{\partial f(y, x, t)}{\partial y_i} \frac{dy_i}{dt}}{\frac{\partial f(y, x, t)}{\partial x_i} \frac{dx_i}{dt}} \quad (13)$$

# Distance Functions



- These concepts actually define a distance function measure of productivity growth.
- Define a measure  $\theta$  for a new technology based on the old technology as

$$D_{\theta}(x, y) = \min_{\theta} [\theta : F(x\theta) \geq y] \quad (14)$$

