

Revenue Functions – Lecture XXIII

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- 1 Univariate Revenue Function
- 2 Multioutput Revenue Function

Univariate Revenue Function

- As a starting point consider the simple univariate revenue function

$$R(p, x) = pf(x_1, x_2, x_3, x_4, x_5) \quad (1)$$

- Formulating this as a translog

$$\ln(R(p, x)) = \alpha_0 + \alpha_1 \ln(p) + A_{11} \frac{\ln(p)^2}{2} + \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 \end{pmatrix} \begin{pmatrix} \ln(x_1) \\ \ln(x_2) \\ \ln(x_3) \\ \ln(x_4) \\ \ln(x_5) \end{pmatrix} \\ + \frac{1}{2} \begin{pmatrix} \ln(x_1) \\ \ln(x_2) \\ \ln(x_2) \\ \ln(x_3) \\ \ln(x_4) \\ \ln(x_5) \end{pmatrix}' \begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} & B_{15} \\ B_{12} & B_{22} & B_{23} & B_{24} & B_{25} \\ B_{13} & B_{23} & B_{33} & B_{34} & B_{35} \\ B_{14} & B_{24} & B_{34} & B_{44} & B_{45} \\ B_{15} & B_{25} & B_{35} & B_{34} & B_{44} \end{pmatrix} \begin{pmatrix} \ln(x_1) \\ \ln(x_2) \\ \ln(x_3) \\ \ln(x_4) \\ \ln(x_5) \end{pmatrix} + \ln(p) \begin{pmatrix} \Gamma_{11} \\ \Gamma_{12} \\ \Gamma_{13} \\ \Gamma_{14} \\ \Gamma_{15} \end{pmatrix}' \begin{pmatrix} \ln(x_1) \\ \ln(x_2) \\ \ln(x_3) \\ \ln(x_4) \\ \ln(x_5) \end{pmatrix} \quad (2)$$

- Equation 2 can be viewed as a second order Taylor series expansion of

$$R(p, x) = p \left[\alpha_0 \prod_{i=1}^5 (1 - \exp(\alpha_i x_i)) \right] \quad (3)$$

basically a Mitcherlich-Buale production function.

- In fact, we could estimate this relationship for the Food and Kindred product dataset from the online session.

Parameter Estimates

Parameter	Estimate	Variance	Parameter	Estimate	Variance
α_0	5421.98	4380.60	B_{15}	-21.88	170.65
α_1	-120.81	27.83	B_{22}	-38.38	137.05
A_{11}	10.79	2.30	B_{23}	-69.25	44.06
β_1	892.37	1060.43	B_{24}	85.78	108.24
β_2	662.16	850.76	B_{25}	-17.42	195.99
β_3	862.39	286.51	B_{33}	-123.10	35.48
β_4	-1272.26	1243.55	B_{34}	12.29	35.96
β_5	-1275.69	1130.78	B_{35}	81.09	41.36
B_{11}	38.93	131.75	B_{44}	53.50	208.90
B_{12}	-42.35	109.98	B_{45}	-1.35	167.91
B_{13}	-10.25	28.07	B_{55}	86.64	299.91
B_{14}	-39.48	158.68			

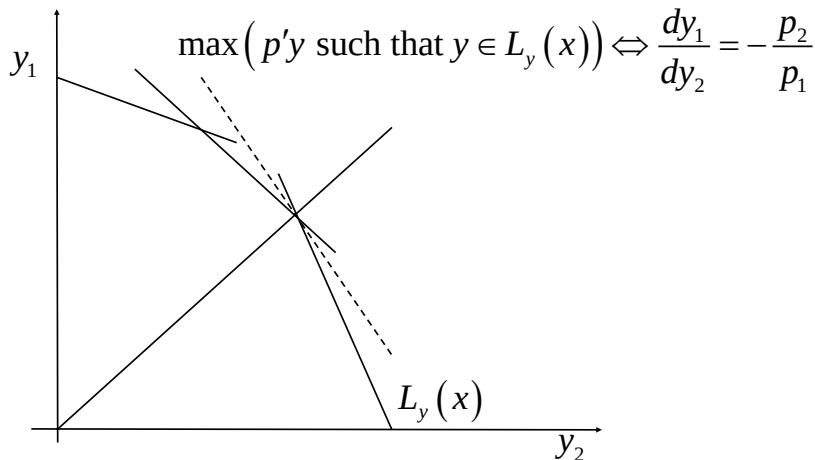


Figure: Product - Product Problem

Mathematics of the Revenue Function

- Given this general formula all of our standard results from duality hold

$$R(p, x) = \max_y (p'y : y \in L_y(x)) \quad (4)$$

- Even the more esoteric disance function formulations

$$D_o(y, x) = \max(\lambda : \lambda y \in L_y(x)) \quad (5)$$

$$R(p, x) = \max_y (p'y : D_o(y, x) = 1)$$

- However, as in the cost function – the most useful results from the revenue function results from applying envelope theory to the indirect function

$$\frac{\partial R(p, x)}{\partial p_i} = y_i(p, x). \quad (6)$$

- Specifically, using the profit function specification

$$\pi(p, w) = \max_x [R(p, x) - w'x] \quad (7)$$

$$\nabla_p \pi(p, w) = p = \nabla_p R(p, x) \ni \nabla_x R(p, x) - w = 0$$

- Thus, consider a simple revenue function

$$\begin{aligned}
 R(p_1, p_2, x_1, x_2) = & \alpha_0 + \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}' \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}' \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \\
 & + \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}' \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' \begin{bmatrix} B_{11} & B_{12} \\ B_{12} & B_{22} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\
 & + \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}' \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}
 \end{aligned}$$

$$y(p, x) = \begin{bmatrix} y_1(p, x) \\ y_2(p, x) \end{bmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} + \begin{bmatrix} \Gamma_{11} & \Gamma_{21} \\ \Gamma_{12} & \Gamma_{22} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (8)$$