

The Stochastic Nature of Production: Lecture VII

Charles B. Moss¹

¹University of Florida

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Stochastic Production Functions

- Just, Richard E. and Rulan D. Pope. 1978. Stochastic Specification of Production Functions and Economic Implications *Journal of Econometrics* 7(1), 67-86.
- Our development of the random characteristics of the production function was largely one of convenience.
- We started with a production function that we wanted to estimate

$$f(x_1, x_2) = \alpha_0 x_1^{\alpha_1} x_2^{\alpha_2}$$
$$g(x_1, x_2) = a_0 + a_1 x_1 + a_2 x_2 + A_{11} x_1^2 + 2A_{12} x_1 x_2 + A_{22} x_2^2 \quad (1)$$

- In order to estimate each function, we multiplied or added a random term to each specification

$$f(x_1, x_2) = \alpha_0 x_1^{\alpha_1} x_2^{\alpha_2} e^u$$

$$\Rightarrow \ln(f(x_1, x_2)) = \tilde{\alpha}_0 + \alpha_1 \ln(x_1) + \alpha_2 \ln(x_2) + u$$

$$g(x_1, x_2) = a_0 + a_1 x_1 + a_2 x_2 + A_{11} x_1^2 + 2A_{12} x_1 x_2 + A_{22} x_2^2 + \nu \quad (2)$$

- Just and Pope discuss three different specifications of the stochastic production functions

$$y = F_1(X) = f(X) e^\epsilon \quad E[\epsilon] = 0$$

$$y = F_2(X) = f(X) \epsilon \quad E[\epsilon] = 1 \quad (3)$$

$$y = F_3(X) = f(X) + \epsilon \quad E[\epsilon] = 0$$

- Each of these specifications has “problematic” implications.
 - For example, the Cobb-Douglas specification implies that all inputs increase the risk of production:

$$\begin{aligned}
 V[f(x_1, x_2)] &= E[(\alpha_0 x_1^{\alpha_1} x_2^{\alpha_2} e^\epsilon)^2] - [E(\alpha_0 x_1^{\alpha_1} x_2^{\alpha_2} e^\epsilon)]^2 \\
 &\Rightarrow \frac{\partial V[f(x_1, x_2)]}{\partial x_1} > 0
 \end{aligned}
 \tag{4}$$

- Note that this expectation is complicated by the fact the expectation of the exponential. Specifically, under log-normal distributions

$$E[e^\epsilon] = e^{\mu + \frac{1}{2}\sigma^2} \tag{5}$$

Just and Pope Propositions

- Just and Pope propose 8 propositions that “seem reasonable and, perhaps, necessary to reflect stochastic, technical input-output relationships.”
 - Postulate 1: Positive production expectations $E[y] > 0$.
 - Postulate 2: Positive marginal product expectations

$$\frac{\partial E[y]}{\partial X_i} > 0 \quad (6)$$

- Postulate 3: Diminishing marginal product expectations

$$\frac{\partial^2 E[y]}{\partial X_i^2} < 0 \quad (7)$$

Just and Pope Propositions - Continued

- Continued
 - Postulate 4: A change in variance for random components in production should not necessarily imply a change in expected output when all production factors are held constant

$$\frac{\partial E[y]}{\partial V[\epsilon]} = 0 \quad (8)$$

- Postulate 5: Increasing, decreasing, or constant marginal risk should all be possibilities

$$\frac{\partial V[y]}{\partial X_i} \gtrless 0 \quad (9)$$

Just and Pope Propositions - Continued

- Continued

- Postulate 6: A change in risk should not necessarily lead to a change in factor use for a risk-neutral (profit-maximizing) producer

$$\frac{\partial X_i^*}{\partial V[\epsilon]} = 0 \quad (10)$$

- Postulate 7: The change in the variance of marginal product with respect to a factor change should not be constrained in sign a priori without regard to the nature of the input

$$\frac{\partial V\left[\frac{\partial y}{\partial X_i}\right]}{\partial X_j} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (11)$$

Just and Pope Propositions - Continued

- Continued
 - Postulate 8: Constant stochastic returns to scale should be possible

$$F(\theta X) = \theta F(X) \quad (12)$$

Individual Functions

- The Cobb-Douglas, transcendental, and translog production functions are consistent with postulates 1, 2, 3, and 8. However, in the case of postulate 5

$$\begin{aligned} E[y] &= f(X) E[e^\epsilon] & V[y] &= f^2(X) V[e^\epsilon] \\ \frac{\partial E[y]}{\partial X_i} &= f_i E[e^\epsilon] & \frac{\partial V[y]}{\partial X_i} &= 2f f_i V[e^\epsilon] \end{aligned} \quad (13)$$

- The marginal effect of input use on risk must always be positive. Thus, no inputs can be risk-reducing.
- For postulate 4, under normality

$$\frac{\partial E[y]}{\partial V[\epsilon]} = \frac{1}{2\sigma} f(X) e^{\frac{\sigma}{2}} > 0 \quad (14)$$

- Thus, it is obvious that our standard specification of stochastic production functions is inadequate.

An Alternative Specification

- Basic formulation for any function

$$\begin{aligned} y &= F_4(X) = f(X) + h(X)\epsilon \\ E[\epsilon] &= 0, \quad V[\epsilon] = \sigma^2 \end{aligned} \tag{15}$$

- Increasing the abstraction slightly

$$\begin{aligned} y_t &= f(Z_t, \alpha) + h(Z_t, \beta)\epsilon_t \\ E[\epsilon_t], E[\epsilon_t^2] &= 1, \quad E[\epsilon_t\epsilon_s] = 0 \quad t \neq s \end{aligned} \tag{16}$$

- In the case of the Cobb-Douglas

$$\begin{aligned} f(Z_t, \alpha) &\equiv (\ln(Z_t))' \alpha = z_t' \alpha \\ h(Z_t, \beta) &\equiv (\ln(Z_t))' \beta = z_t' \beta \\ Z_t &= Z(X_t) \end{aligned} \tag{17}$$

Consistent Estimation

- Rewriting the error term

$$u_t = h(Z_t, \beta) \epsilon_t \quad (18)$$

- So the production function can be rewritten as

$$y_t = f(Z_t, \alpha) + u_t \quad E(u_t) = 0 \quad (19)$$

- Where the disturbances are heteroscedastic.
- Under appropriate assumptions, a nonlinear least-squares estimate of this expression yields consistent estimates of α .
- Thus, these estimates can be used to derive consistent estimates of u_t

$$\hat{u}_t = y_t - f(Z_t, \hat{\alpha}) \quad (20)$$

Heteroscedasticity

- Consistent estimates of β are obtained in the second stage by regressions on u . Following the method suggested by Hildreth and Houck

$$\hat{u}_t^2 = h^2(Z_t, \beta) \quad (21)$$

Panel Data Specification

- Going back to the simultaneity specification

$$Y = Ax_1^\alpha x_2^\beta e^{u_0^+} \quad (22)$$

- This expression becomes

$$\begin{aligned} \ln(y) - \alpha \ln(x_1) - \beta \ln(x_2) &= \ln(A) + u_1^+ + \epsilon_1 \\ \ln(y) - \ln(x_1) &= \ln(P) - \alpha - \ln(W_1) + u_2^+ + \epsilon_2 \\ \ln(y) - \ln(x_2) &= \ln(P) - \beta - \ln(W_2) + u_3^+ + \epsilon_3 \end{aligned} \quad (23)$$

- In order to discuss this specification, we will begin with a brief survey of estimation using panel data.
- As a starting point of this model, we consider a panel regression

$$y_{it} = \alpha + \beta x_{it} + \epsilon_{it} \quad \begin{array}{l} i = 1, 2, \dots, N \\ t = 1, 2, \dots, T \end{array} \quad (24)$$

- This specification is implicitly pooled, the value of the coefficients are the same for each individual at every point in time.
- As a starting point, we consider generalizing this representation to include differences in constant of the regression that are unique to each firm

$$y_{it} = \alpha_i + \beta x_{it} + \epsilon_{it} \quad (25)$$

- This specification can be expanded further to allow for differences in the slope coefficients across firms

$$y_{it} = \alpha_i + \beta_i x_{it} + \epsilon_{it} \quad (26)$$

- Based on these alternative models, we conceptualize a set of nested tests.
 - First we test for overall pooling (i.e., the production function have the same constant and slope parameters for every firm).
 - If pooling is rejected for both sets of parameters, we hypothesize that the constants differ for each firm, while the slope coefficients are the same.
 - Next, consider a random specification for the individual constants

$$y_{it} = (\bar{\alpha}_i + \nu_t) + \beta x_{it} + \epsilon_{it} \quad (27)$$

- Hsiao, Cheng. 1986. *Analysis of Panel Data* New York: Cambridge University Press.