

Estimation of the Primal Production Function: Lecture V

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September 4, 2018

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Ordinary Least Squares

- The most straightforward concept in the estimation of production function is the application of ordinary least squares.
- Starting with the simple quadratic production function

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + A_{11}x_1^2 + A_{12}x_1x_2 + A_{13}x_1x_3 + A_{22}x_2^2 + A_{23}x_2x_3 + A_{33}x_3^2 \quad (1)$$

Note that we have already imposed symmetry on the quadratic. From an estimation perspective since $x_1x_2 = x_2x_1$ any other approach would not work.

- Using the data from Indian and Illinois, we apply ordinary least squares to this specification.

Empirical Estimates of the Quadratic

Parameter	Estimate	Parameter	Estimate
a_0	100.2819 (6.8117)	A_{12}	-0.00089 (0.0030)
a_1	-0.0054 (0.0791)	A_{13}	0.00073 (0.00047)
a_2	0.2344 (0.1063)	A_{22}	0.00019 (0.00038)
a_3	0.0590 (0.0795)	A_{23}	-0.00087 (0.00049)
A_{11}	0.000138 (0.00030)	A_{33}	-0.00007 (0.00031)

- Do these estimates make any sense? What is wrong?

The Cobb-Douglas Production Function

- Turning to the Cobb-Douglas form

$$y = Ax_1^\alpha x_2^\beta x_3^\gamma$$

$$\Rightarrow \ln(y) = \ln(A) + \alpha \ln(x_1) + \beta \ln(x_2) + \gamma \ln(x_3)$$

$$\Rightarrow \ln(y) = \alpha_0 + \alpha_1 \ln(x_1) + \alpha_2 \ln(x_2) + \alpha_3 \ln(x_3) + \epsilon \quad (2)$$

- What are some of the problems with this specification?
 - First, the input level may be zero.
 - What is the production theoretic problem with zero inputs levels?
 - What is the econometric problem with zero input levels?
 - Second, what is the assumption about the error term?

Empirical Estimates of the Cobb-Douglas Function

Parameter	Estimate
α_0	4.5858*** (0.0561)
α_1	0.0126 (0.0118)
α_2	0.1680** (0.0073)
α_3	0.0132** (0.0063)

Transcendental Production Function

- The transcendental production function has many of the same problems as the Cobb-Douglas
- The productin function can be written as

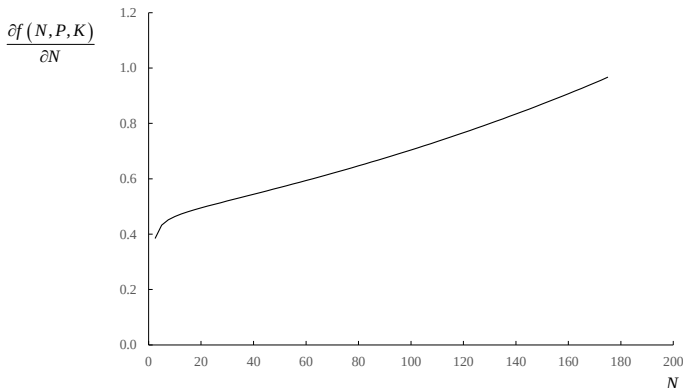
$$y = Ax_1^{a_1} e^{b_1 x_1} x_2^{a_2} e^{b_2 x_2} x_3^{a_3} e^{b_3 x_3}$$
$$\ln(y) = a_0 + a_1 \ln(x_1) + b_1 x_2 + a_2 \ln(x_2) + b_2 x_2 + \quad (3)$$
$$a_3 \ln(x_3) + b_3 x_3 + \epsilon$$

- Again, what are the assumptions about zeros or the distribution of the error terms.

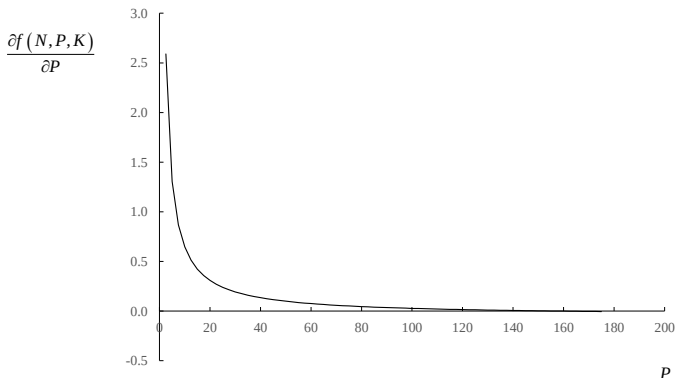
Empirical Results of the Transcendental

Parameter	Parameter	Parameter	Parameter
a_0	4.5520 (0.05875)	b_2	-0.00004 (0.00045)
a_1	-0.000006 (0.01564)	a_3	0.0025 (0.0090)
b_1	-0.00039 (0.00038)	b_3	0.00080 (0.00047)
a_2	0.0166 (0.00887)		

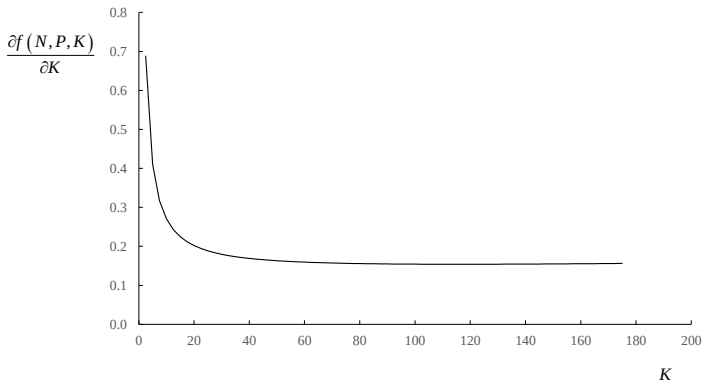
Marginal Product of Nitrogen



Marginal Product of Phosphorous



Marginal Product of Potash



Nonparametric Production Functions

- It is clear from our discussions on production functions that the choice of production function may have significant implications for the economic results from the model.
 - The Cobb-Douglas function has linear isoquants that has implications for the input demand functions
 - While the Cobb-Douglas function has no stage III, the quadratic production function is practically guaranteed a stage III.
- Thus, one approach is to generate nonparametric functional forms.
 - These nonparametric functional forms are intended to impose allow for the maximum flexibility in the input-output map.
 - The approach is different that the nonparametric production function suggested by Varian.

Fourier Expansion

$$f(x) = \beta_0 + \beta'x + \frac{1}{2}x'Bx + \sum_{\alpha=1}^A \left[\beta_{0\alpha} + \sum_{j=1}^j (\beta_{j\alpha} \cos(jk'_\alpha x) + \gamma_{j\alpha} \sin(jk'_\alpha x)) \right] \quad (4)$$

$$B = - \sum_{\alpha=1}^A \beta_{0\alpha} k_\alpha k'_\alpha$$

Nonparametric Regressions

- A nonparametric regression is basically a moving weighted average where the weights of the moving average change for various input levels.

$$\hat{y}(x) = \int_{-\infty}^{\infty} y(z) f(y, z, x, \delta) dz. \quad (5)$$

- In this case $y(x)$ is the estimated function value conditioned on the level of inputs x .
- The value $y(z)$ is the observed level of the observed input z .
- $f(y, z, x, \delta)$ is a kernel function which weights the observations based on a distance from the point of approximation.

Kernel

- In this application, I use a Gaussian kernel.

$$f(y, z, x, \delta) = \frac{1}{\sqrt{2\pi}} \delta^{-1} \exp \left[-\frac{(z - x)^2}{2\delta^2} \right] \quad (6)$$

- The multivariate form of the Gaussian kernel function is expressed as

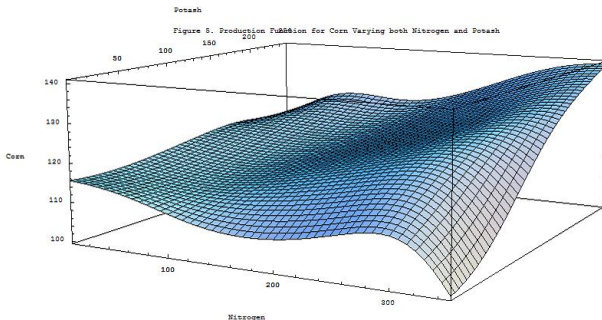
$$f(y, \{z\}, \{x\}, A, \delta) = \frac{1}{\sqrt{2\pi}} \delta^{-1} |A|^{-1/2} \exp \left[-\frac{1}{\delta} (z - x)' A^{-1} (z - x) \right] \quad (7)$$

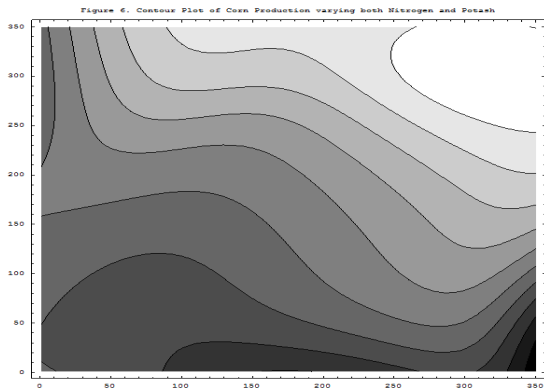
- Because of the discrete nature of the expansion, I transform the continuous distribution into a discrete Gaussian distribution

$$w(y, \{x_i\}, \{x\}, A, \delta) = \frac{f(y, \{x_i\}, \{x\}, A, \delta)}{\sum_{j=1}^N f(y, \{x_j\}, \{x\}, A, \delta)} \quad (8)$$

- The estimated value of the production function at point x can then be computed as

$$\hat{y}(x) = \sum_{i=1}^N w(y, \{x_i\}, \{x\}, A, \delta) y_i \quad (9)$$





Marginal Physical Product of Nitrogen

Figure 4. Distribution of the Marginal Physical Product of Nitrogen

