

Introduction to Mathematical Programming and Course Overview

Lecture I

- I. Course Overview
- II. Overview of Mathematical Programming in Economic Analysis

A. Basic Microeconomic Problems

- 1. The Consumer's Problem

$$\max U(\underline{x})$$

$$s.t. \underline{x}p' \leq Y$$

where $U()$ is the consumer's utility function, $\underline{x}$ is a vector of goods, p is a vector of prices, and Y is the consumer's income. What does this mean? How do we determine the optimal? Is this specification sufficient?

- 2. The Producer's Problem

$$\max \underline{p}y' - \underline{w}x'$$

$$s.t. y = F(\underline{x})$$

where p is a vector of output prices, y is a vector of output levels, w is a vector of input prices, x is a vector of input levels, and $F()$ is vector-valued production function. Again, is this problem sufficient?

- 3. Both of these problems from microeconomic theory imply finding an extreme value to an objective function.

B. "Typical" and "Non-Typical" applications of Mathematical Programming

- 1. Reading

Day, R. H. and E. Sparling. "Optimization Models in Agricultural and Resource Economics."

Day, R. H. "On Economic Optimization: A Non-Technical Survey."

Moss, C. B. "Applied Optimization in Agriculture." In *Handbook of Applied Optimization*

- 2. Day and Sparling categorize optimization studies in agriculture and resource economics into seven categories:

- a. Food and Diet
- b. Farm and Agribusiness Management
- c. Farm Firm Development (Farm growth models)
- d. Production Response
- e. Interregional and Spatial Economics
- f. Natural Resources
- g. Agricultural Development

- 3. To Day and Sparling's list I add Econometric Estimation and Policy Analysis and Development.

C. A Brief Explanation of Topics

- 1. Food and Diet—The food and diet research can be characterized by two major focuses

- a. Least cost combinations of foods to meet dietary needs. Stigler's "cost of subsistence."
 - b. Least cost feed ration studies.
- 2. Farm and Agribusiness Management—The scope of the research into this area is so vast that only a brief review is offered here.
 - a. Initially, linear programming was used to find optimal crop mix and proportion. This work has grown into large extension farm planning efforts such as OK Farms. Some of these applications now use integer programming.
 - b. Extending the machinery section and integrating a large machinery and labor complement led to B-10 in Indiana which is essence is a machinery planning model.
 - c. Agribusiness firms benefit from optimization techniques in several ways:
 - (i) Inventory Management
 - (ii) Transshipment Models
 - (iii) Least Cost Feeds
 - d. Replacement models based on Bellman's principle of optimality.
 - e. Risk programming
- 3. Farm Firm Development—The typical farm firm development model is primarily interested in firm growth.
 - a. Multiperiod programming allowed researchers to examine questions related to investment, capital accumulation, and debt-choice.
 - (i) Swanson, and Loftsgard and Heady focused on the managerial or production response.
 - (ii) Later Irwin and Baker, and Martin and Plaxico focused on the financial aspects of growth.
 - (iii) Moss 1984 focused on the effect of tax codes on firm growth while Moss 1987 focused on the effect of exogenous factors (macroeconomic policy).
 - (iv) Early studies in this area were deterministic, obviously excluding an important aspect of firm growth. Johnson, Tefertiller and Moore opened the door by allowing crop yields to be stochastic. Moss 1987 drew from other techniques to approximate stochastic optimal control.
 - b. A huge problem with these studies has typically been the size of the matrix.
- 4. Production Response—Production response models have been used to study the impact of some policy or external shock to the sector.

- a. From the firm level, the effect of changing fertilizer prices, labor availability, or support prices on firm outputs, profits, and input demands could be mapped out—much like the duality approach in production.
 - b. The firm level effects were then aggregated to the sector level.
 - c. CARD.
- 5. Interregional and Spatial Economics—A large section of the literature, but I only want to discuss two types:
 - a. Arbitrage or regional supply and demand models. If demand were distributed identically across all areas of the country, but there was a primary supply region—say orange juice in Florida. What would the price equilibrium look like? What is the impact on profit of a change in transportation cost? What happens if Brazil enters into the market?
 - b. Plant location models. In the past most heavy industry was located in the north, particularly in the area around the great lakes. As the population shifted to the south, more industry shifted to the south. Why? Current topic, the technology is being developed to condense most of the water out of milk to reduce the shipping cost. Will this cause the dairies to move to Wisconsin? If you were John Deere looking at where to locate a warehouse, how would you do it?
- 6. Natural Resources—Again, several problems have been undertaken, a few are:
 - a. Exhaustible resources
 - b. Commercial fisheries
 - c. Aquifer management
 - d. Reservoir management
 - (i) Chance constrained programming
 - (ii) Spatial allocation
 - e. Pollution
- 7. Agricultural Development
 - a. Allocation of resources (debt, labor, foreign exchange, etc.) between agriculture and other sectors.
 - b. Farm level development.
- 8. Econometric Estimation—Over the past two decades econometrics has shifted increasingly to maximum likelihood techniques as opposed to ordinary least squares (OLS) and generalized least squares (GLS). Typically, most of the OLS and GLS applications had nice “closed form” solutions, or the solution of the least squares problem could be written down

$$b = (x'x)^{-1} x'y$$

However, more recently estimation has relied on numerical solutions. For example in the autoregressive conditional heteroscedasticity (ARCH) model

$$\max_{a,b} \sum_{t=1}^T \frac{T}{2} \left[-\ln \left(\frac{1}{a_0 + a_1 x_{t-1}^2 + a_2 x_{t-2}^2} \right) - \frac{(y_t - b y_{t-1})^2}{a_0 + a_1 x_{t-1}^2 + a_2 x_{t-2}^2} \right]$$

s.t. $x_t = y_t - b y_{t-1}$

9. Policy Analysis

- a. Targeted Tariff/Subsidies in Trade.
- b. General equilibrium budgetary analysis.