

Lecture VIII: Finish State Preference Theory, Start Efficient Markets

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I. Fisher Separation and Capital Budgeting

- A. One very powerful result of deterministic financial analysis is Fisher's Separation Theorem that suggests that it is appropriate to discount at the market interest rate or weighted average cost of capital.
 - 1. In a Robinson Crusoe economy investment is determined by the tangency between the production possibility frontier and the individual's utility function (Figure 1). This equilibrium endogenizes the choice of investment.
 - 2. Unfortunately, the concept of the project level discount rate is more complicated than the Robinson Crusoe Economy. One alternative is to posit frictionless capital markets in which investors trade capital (literally they trade consumption later for consumption now by loaning money). This scenario changes the investment choice to the scenario depicted in Figure 2.
 - a. The line in Figure 2 represents a market for capital with slope of $1/(1+r)$.
 - b. The introduction of the capital market separates the consumption and investment decision at the micro-level.
 - c. However, in aggregate the market must clear.
- B. The question then becomes: How is the Fisher Separation affected by risk?

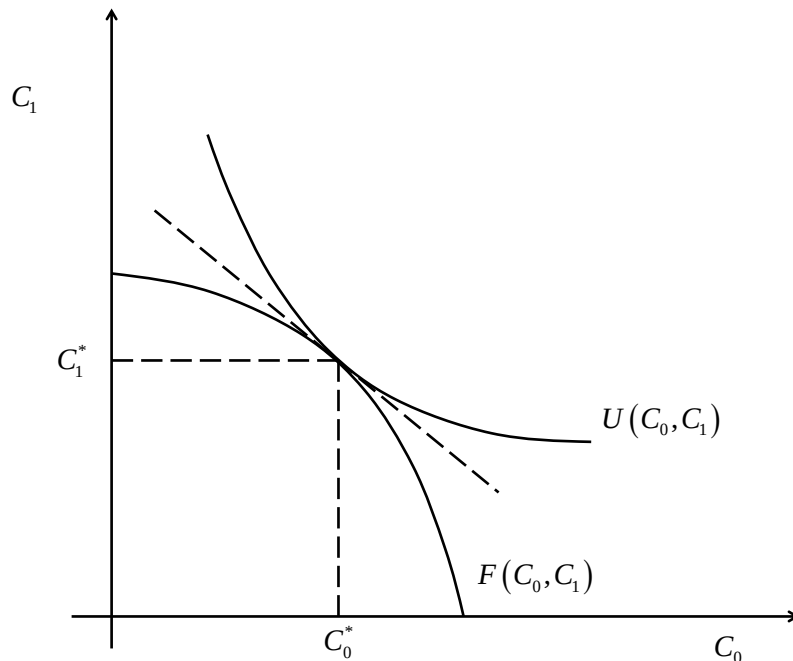


Figure 1: Robinson Crusoe Capital Market

1. It can be shown that firms maximizing the current price of their shares also maximize the current shareholder's expected utility if the capital market is
 - a. Perfectly competitive and frictionless, and
 - b. Complete.
2. The two conditions of perfect and complete capital markets ensure that the prices of a full set of pure securities can be obtained from the prices of the market securities.
3. The firm's objective function then becomes

$$\sum_s Q_{js} P_s \quad (1)$$

where Q_{js} is defined as the state s contingent end-of-period payoff on firm j 's existing securities. In this formulation the price of a firm's current share is determined by

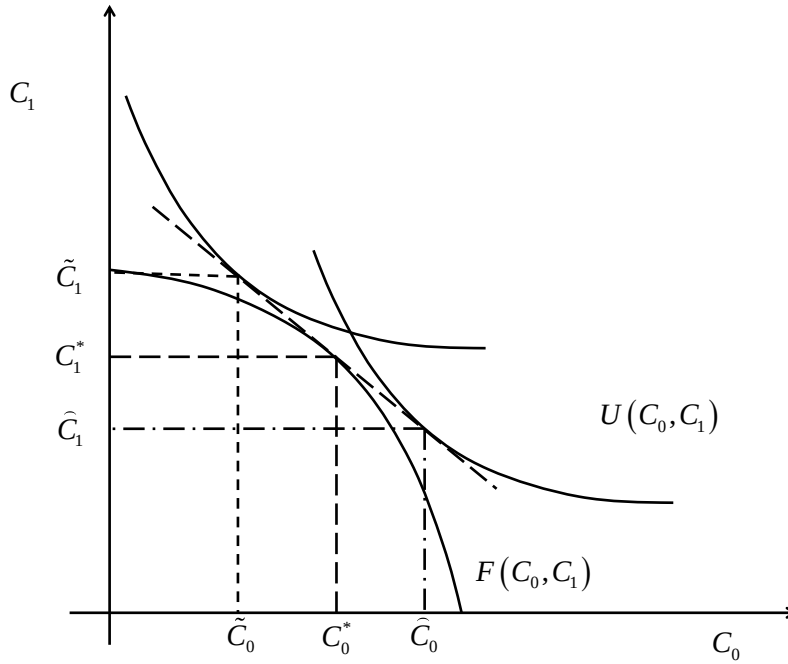


Figure 2: Capital Market with Trade

- a. The firm's state contingent production function $Q_{js} = \phi_j(I_j, s)$ which transforms current resources into state contingent future payoffs, and
- b. The initial investment I_j which represents the current cost to the firm of producing that state-contingent payoff.
4. It follows that the price Y_j for which the current owners could sell the firm prior to making the investment I_j is

$$Y_j = \sum_s Q_{js} P_s - I_j \quad (2)$$

where the firm's technology constraints are captured in its production function

$$Q_{js} = \phi_j(I_j, s). \quad (3)$$

For infinitely divisible investments

$$\frac{dY_j}{dI_j} = \sum_s P_s \phi'_j(I_j, s) - 1 = 0. \quad (4)$$

C. Discrete Applied Case

1. Assume two firms have two different investment opportunities presented in Table 1.

Table 1: Different Investment Opportunities

Firm A – Price = 62 - Proposed Price = 10		
State	Stock	Proposed
State 1	100	10
State 2	30	12
Firm B – Price = 56 - Proposed Price = 8		
State	Stock	Proposed
State 1	40	12
State 2	90	6

2. Thus, the payoff for Firm A (equal to its market price) is

$$100P_1 + 30P_2 = 62 \quad (5)$$

while the payoff for Firm B (equal to its market price) is

$$40P_1 + 90P_2 = 56. \quad (6)$$

Solving for the price for each state

$$\begin{aligned} \begin{bmatrix} 100 & 30 \\ 40 & 90 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} &= \begin{bmatrix} 62 \\ 56 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} &= \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}. \end{aligned} \quad (7)$$

3. Computing the Net Present Values

$$\begin{aligned} NPV_A &= 10(0.5) + 12(0.4) - 10 = -0.2 \\ NPV_B &= 12(0.5) + 6(0.4) - 8 = 0.4 \end{aligned} \quad (8)$$

Accept investment B , reject investment A .

II. Efficient Capital Markets

- A. As eluded to in the previous section, the purpose of a capital market is to facilitate the flow of funds between borrowers and lenders.
1. The capital market serves to allocate capital between competing uses in a way that maximizes economic welfare. This is accomplished much the same way we think about traditional factor markets. The excess capital is exhausted from the highest marginal value product to the lowest. At each increment a valid decision is whether the supplier values the increment in capital more than the payoff. If such an equilibrium occurs, the market is said to be allocatively efficient.
 2. The concept of allocative efficiency is typically contrasted with the concept of perfect capital markets. The conditions necessary for perfect capital markets are
 - a. Markets are frictionless (i.e., there are no transaction costs or taxes) all assets are perfectly divisible and marketable, and there are no constraining regulations.
 - b. There is perfect competition in product and security markets.
 - c. Markets are informationally efficient (i.e., information is costless) and it is received simultaneously by all individuals.
 - d. All individuals are rational expected utility maximizers.
 3. Given these conditions, the capital market will be both allocative and operationally efficient. Operational efficiency deals with the costs of transferring funds.
 4. Capital market efficiency is much less restrictive than the notion of perfect capital markets outlined above. In an efficient capital market prices fully and instantaneously reflect all available relevant information. This means that when assets are traded prices are accurate signals for capital allocation.
 5. Capital markets are operationally efficient if intermediaries who provide the service of channeling the funds from savers to borrowers do so at the minimum cost that provides them a fair return for their services.

- B. Fama defines three different types of efficiency based on different notions of all relevant information.
1. *Weak-Form Efficiency*: No investor can earn excess return by developing trading rules based on historical price and return information.
 2. *Semistrong-Form Efficiency*: No investor can earn excess returns from trading information based on any publically available information.
 3. *Strong-Form Efficiency*: No investor can earn excess returns using information whether publically available or not.
- C. The value of information
1. Mathematically, the value of information is defined as

$$V(\eta) \equiv \sum_m q(m) \max_a \sum_e p(e|m) U(a, e) - V(\eta_0) \quad (9)$$

- a. Breaking this down, there are m different outcomes for the information set. For example, suppose your information was to poll 10 people regarding the availability of capital in the economy. There are 11 outcomes to the poll. None may experience credit restrictions, one may have encountered credit restrictions, and so forth.
- b. The primary proposition to the value of information is that knowing the value of the survey changes the relative likelihood of the outcomes. For example, as depicted in Table 2 no people in the poll reported being credit constrained. Based on this result, we may conjecture that the 0.01 of the population is credit constrained, 0.25 percent of the population have adequate credit, and 0.75 percent have access to abundant credit. In this case, we say that the poll is informative.
- c. This change in probability structure may then have an affect on some management strategy. For example, as presented in Table 3, if no people said that credit was constrained your new probability structure would tilt toward building a new factor because you have a greater

Table 2: Poll Results

Credit Constrained	0 People in Report Constraint	10 People in Report Constraint
Credit is Constrained	0.01	0.75
Credit is Adequate	0.24	0.24
Credit is Abundant	0.75	0.01

probability of earning \$ 2,000. However, if all 10 people survey indicate tight credit, the decision maker would choose to update the old factory. Mathematically, the choice is determined by

Table 3: Investment Payoff

Strategy	Credit Condition		
	Abundant	Adequate	Constrained
Build new factory	2,000	750	-500
Update old factory	1,000	500	250

$$\max_a \sum_e p(e|m) U(a, e) \quad (10)$$

Given the new probability structure, selection the action a that maximizes the expected payoff.

- d. The value of the information is the difference between the expected payoff under the information and the expected payoff without the additional information.

D. From Fama, E.F. 1976. *Foundations of Finance* New York: Basic Books, Inc. – Chapter 5.

1. Given a sequence of discrete points in time $t, t + 1, \dots$ let
 - ϕ_{t-1} = the set of all available information at time $t - 1$ which is relevant for determining security prices at time $t - 1$.
 - ϕ_{t-1}^M = the set of information that the market uses to determine security prices at $t - 1$. Thus, ϕ_{t-1}^M is a subset of ϕ_{t-1} .

- $p_{j,t-1}$ = the price of security j at time $t - 1$ where $j = 1, \dots, n$.
 - $f_m(p_{1,t+\tau}, \dots, p_{n,t+\tau} | \phi_{t-1}^M)$ = the joint probability function for prices at $t + \tau$ ($\tau \geq 0$) assessed by the market at time $t - 1$ on the basis of ϕ_{t-1}^M .
 - $f(p_{1,t+\tau}, \dots, p_{n,t+\tau} | \phi_{t-1})$ = the "true" joint density function for security prices at time $t + \tau$ (τ) implied by ϕ_{t-1} .
2. Thus, on the basis of ϕ_{t-1}^M the market assesses the joint distribution of security prices at time $t - f_m(p_{1,t}, \dots, p_{n,t} | \phi_{t-1}^M)$. From this assessment the market returns the appropriate security prices $p_{1,t-1}, \dots, p_{n,t-1}$ are determined that clear the securities market.
 3. From the market clearing idea we get the efficient market conditions

$$f_m(p_{1,t}, \dots, p_{n,t} | \phi_{t-1}^M) = f(p_{1,t}, \dots, p_{n,t} | \phi_{t-1}) \quad (11)$$

4. Unfortunately, the model as currently specified because of the generalized form lacks a detailed linkage between $f_m(p_{1,t}, \dots, p_{n,t} | \phi_{t-1}^M)$ and $p_{1,t-1}, \dots, p_{n,t-1}$. Some model of market equilibrium is required. This is a central problem because any test of market efficiency is a combination of two tests. One test of the specification of the equilibrium and the second one is a test for market efficiency.