

Lecture XXIII: Contracts and Asymmetric Information

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Readings for next time

Limited Liability and Contracts

Readings for Monday

- ▶ Erickson, K., A.K. Mishra, and C.B. Moss. 2003. Cash Rents, Imputed Returns, and the Valuation of Farmland Revisited. *Government Policy and Farmland Markets*. Iowa State University Press, pp. 223-235.
- ▶ Featherstone, A.M. and C.B. Moss. 2003. The Certainty Equivalence of Farmland Values: 1910 to 2000. *Government Policy and Farmland Markets*. Iowa State University Press, pp. 159-178.
- ▶ Moss, C.B. 1997. Returns, Interest Rates, and Inflation: How They Explain Changes in Farmland Values. *American Journal of Agricultural Economics* 79, 1311-1318.
- ▶ Moss, C.B. and A. Katchova. 2005. Farmland Valuation and Asset Performance. *Agricultural Finance Review* 65, 119-130.

Limited Liability and Contracts

- ▶ Innes, R.D. 1990. Limited Liability and Incentive Contracting with Ex-Ante Action Choices. *Journal of Economic Theory* 52, 45-67.
- ▶ This group of literature follows on the basic concepts of Stiglitz and Weiss, but allows us to bring out certain concepts important to the agricultural credit market.
- ▶ This first piece looks at the question of designing an ownership contract in the principal-agent problem. The basic concept is that the debt contract is "optimal".

- ▶ Motivated by the infeasibility of a fixed payment, the following analysis develops an entrepreneur's financial contract choice problem wherein
 - (1) investors (c.f., principals) are competitive, simply requiring that they receive a market-determined "fair" return on their investment;
 - (2) given the financial contract, the entrepreneur (c.f., agent) makes an effort choice that the investors cannot observe and, thus, that cannot be specified in the contract; and
 - (3) the entrepreneur chooses the investor payoff function (c.f. fee schedule) to maximize his expected utility (of profit or effort) subject to
 - (i) limited liability restrictions,
 - (ii) the investor expected payoff requirement, and
 - (iii) his own effort choice responses.

- ▶ "The latter 'monotonic contract' constraint can be motivated either by a requirement that investors never have an incentive to sabotage the firm or by an ability of entrepreneurs to costlessly revise their profit reports upward (with hidden borrowing for example).

The Model

- ▶ A risk-neutral agent is endowed with a technology that produces a one-time payoff $\pi \in [0, \infty)$ (i.e., profit).
- ▶ This payoff requires two inputs - investment funds and entrepreneurial effort (e). The investment is obtained by outside investors $I > 0$.
- ▶ The cumulative density function for the return is $G(\pi | e)$ and the probability density function is denoted $g(\pi | e)$ – the distribution of the payoff is a function of the effort the farmer puts in.

- ▶ Higher levels of effort produce a higher probability a higher payoff

$$\frac{\partial}{\partial \pi} \left(\frac{g_e(\pi | e)}{g(\pi | e)} \right) > 0 \quad (1)$$

for all $e > 0$ and $\pi \geq 0$. Also $E(\pi | e = 0) = 0$.

- ▶ Investors in the entrepreneurial firm are risk-neutral and require an expected return equal to that on a risk-free bond (ρ).
- ▶ Investors cannot observe e , but they can infer it from the entrepreneur's effort from his utility maximization choice.
- ▶ Thus, they invest I to obtain a required rate of return ρ based on the terms of the contract.

- ▶ The basic concept is to design a payoff function $B(\pi)$.
- ▶ Liability limits constrain this function in two ways: (1) $B(\pi) \leq \pi$; the entrepreneur cannot be required to pay more than the profits available to him; and (2) $B(\pi) \geq 0$; the investor's liability is limited to his investment in the firm.
- ▶ The entrepreneur's utility function is specified as

$$V(w, e) = a(e)w - v(e) \quad (2)$$

- where $w = \pi - B(\pi)$ is the payoff function – profit less that amount paid to the investor,
- $a(e) > 0$ implies a positive utility from the payoff, and
- $-v(e) < 0$ denotes disutility of effort.

- Given a fixed payoff function $B(\pi)$ the entrepreneur chooses his effort by

$$\max_{e \geq 0} E \{ V(\pi - B(\pi), e) | e \} = a(e) \int_0^{\infty} (\pi - B(\pi)) g(\pi | e) d\pi - v(e) \quad (3)$$

- **COROLLARY 1.** For all $B(\pi)$ functions satisfying the liability constraints $0 \leq B(\pi) \leq \pi \forall \pi$, (a) $E\{V(\pi - B(\pi), e) | e\}$ is bounded above and below on the domain $e \in [0, e_{max}]$, and therefore (b) there exists at least one solution to Equation 10.

$$\begin{aligned} \max_{B, e} & a(e) \int_0^\infty (\pi - B(\pi)) g(\pi | e) d\pi - v(e) \\ \text{s.t. } & \int_0^\infty B(\pi) g(\pi | e) d\pi \geq (1 + \rho) I \\ & e \text{ solves 3} \\ & 0 \leq B(\pi) \leq \pi \forall \pi \end{aligned} \tag{4}$$

- Next, Innes considers that $B(\pi)$ is nondecreasing

$$B(\pi + \epsilon) \geq B(\pi) \quad \forall (\pi, \epsilon) \in R_+^2. \quad (5)$$

- (1) After observing a perfect signal of firm profits, investors may be in a position to sabotage the firm, essentially burning as much of these profits as they choose.
- (2) Alternatively, the entrepreneur may observe a perfect signal of firm profits slightly before they are realized, although investors can only observe total net cash flows of the firm on the date profits are realized. In this case, the entrepreneur could not alter firm profits ... but he could supplement these inflows with costless borrowings, $M > 0$, revealing an apparent profit of $\pi^* = \pi + M$ to investors.

Optimal Monotonic Contract

- ▶ Optimal Monotonic Contract – The primary concept is to justify why debt is the 'best' contract.
- ▶ Consider an alternative monotonic contract $B^{ND}(\pi)$ in Figure 14. Essentially the non-debt contract would be an equity investment of some sort.

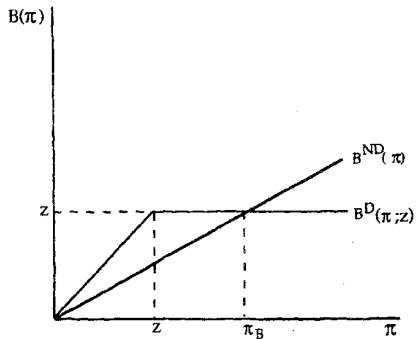


FIGURE 1

- ▶ "It then shows that by moving to a debt contract that yields investors the same expected return as $B^{ND}(\pi)$, the entrepreneur will commit himself to a higher effort level."
- ▶ "Finally, since the debt-contract effort is still **less** than the "first-best" choice, it is shown that the entrepreneur will prefer the debt contract to the nondebt contract, contradicting the supposition that $B^{ND}(\pi)$ was optimal.
- ▶ **ASSUMPTION 3:** When $B(\pi)$ takes a standard debt form, $B(\pi) = B^D(\pi, z) \equiv \min(\pi, z)$, $z > 0$, there is a unique solution to the entrepreneur's effort choice problem (Equation 10). This solution will be denoted by $e^D(z)$.

- Standard fare for this type of analysis – assume that $B^{ND}(\pi)$ and $e^{ND} > 0$ exist. $B^{ND}(\pi)$ must be different from $B^D(\pi)$ in a meaningful way – more than a trivial point. Assume that the expected value of this non-debt function exists

$$Q \equiv E \left\{ B^{ND}(\pi) \middle| e^{ND} \right\} \quad (6)$$

Next construct a debt instrument with the same payoff

$$B^D(\pi; z_0) = \min(\pi, z_0) \Rightarrow E \left\{ B^D(\pi; z_0) \middle| e^{ND} \right\} = Q. \quad (7)$$

– the investor makes the same return under either contract.

- Note that, since $B^{ND}(\pi)$ is monotonic, there is a critical profit level $\pi_B > 0$, such that $B^{ND}(\pi) \leq B^D(\pi; z_0) \forall \pi < \pi_B$... and $B^{ND}(\pi) \geq B^D(\pi; z_0) \forall \pi > \pi_B$ This characterization leads into another lemma which relates the entrepreneur's effort choice under $B^D(\pi; z_0)$ to that under $B^{ND}(\pi) \neq B^D(\pi; z_0)$.

- **LEMMA 1:** Suppose two payoff functions, $B_0(\pi)$ and $B_1(\pi)$, satisfy the following inequalities for some $\pi_B > 0$: (i) $B_0(\pi) \leq B_1(\pi) \forall \pi \leq \pi_B$ (with strict inequality on a set of positive measure), and (ii) $B_0(\pi) \geq B_1(\pi) \forall \pi \geq \pi_B$ (with strict inequality on a set of positive measure). Then

$$\frac{\partial E\{B_1(\pi) - B_0(\pi) | e\}}{\partial e} < 0 \quad (8)$$

if either

- (a) $E\{B_1(\pi) - B_0(\pi) | e\} = 0$, or
- (b) $E\{B_1(\pi) - B_0(\pi) | e\} \leq 0$ and $B_1(\pi) - B_0(\pi)$ is monotone nonincreasing in $\pi \forall \pi > \pi_B$.

- ▶ **LEMMA 2:** $e^D(z_0) > e^{ND}$.
- ▶ "Lemma 2 indicates that the $B^D(\pi; z_0)$ contract yields higher effort than $B^{ND}(\pi)$
 - a. A move from the monotonic nondebt contract, $B^{ND}(\pi)$, to the debt contract, $B^D(\pi; z_0)$, increases the entrepreneur's obligations in low profit states and decreases them in high profit states.
 - b. Loosely speaking, increased effort shifts probability weight to the high profit states, implying that the debt contract gives the entrepreneur more of the benefit from increased effort than does the nondebt contract.

- ▶ Recalling from Equation 1

$$\frac{\partial}{\partial \pi} \left(\frac{g_e(\pi | e)}{g(\pi | e)} \right) > 0 \quad (9)$$

this is referred to as the **monotone likelihood ration property (MLRP)**

- ▶ "The MLRP implies that, for any given profit level, increased effort leads to relatively greater probability weight on all higher levels of profit." Note from Equation 10

$$\begin{aligned} \max_{e \geq 0} E \{ V(\pi - B(\pi), e) | e \} &= a(e) \int_0^\infty (\pi - B(\pi)) g(\pi | e) d\pi - v(e) \\ \Rightarrow \frac{E \{ \min(\pi, z) | e \}}{\partial e} &= - \int_0^z G_e(\pi | e) d\pi > 0 \end{aligned} \quad (10)$$

"where the equality is obtained with integration by parts."

- ▶ "Given Equation 10, Lemma 2 implies that investors' expectations on the $B^D(\pi; z_0)$ contract, $E\{B^D(\pi; z_0) | e^D(z_0)\}$, is greater than Q , the expected investor payoff on $B^{ND}(\pi)$ Moreover, since $e^D(z_0)$ is chosen optimally, the entrepreneur prefers $(B^D(\pi; z_0), e^D(z_0))$ to $(B^D(\pi; z_0), e^{ND})$. Thus, since the entrepreneur is indifferent between $(B^D(\pi; z_0), e^{ND})$ and the proposed nondebt solution $(B^{ND}(\pi), e^{ND})$, the following lemma holds
- ▶ **LEMMA 3:** Any solution to 4 (subject to the constraints) contains a debt contract.

- ▶ **ASSUMPTION 4:** $\exists z : E \{ B^D(\pi; z_0) | e^D(z) \} \geq (1 + \rho) I.$
- ▶ **PROPOSITION 1:** Subject to the monotonic contract constraint (Equation 5) a solution to problem Equation 4 exists and has the following properties:
 - a. $B(\pi) = B^D(\pi; z) \equiv \min(\pi, z), z > 0$ (from Lemma 3);
 - b. $E \{ B(\pi) | e \} = (1 + \rho) I$; and
 - c. $e < e^* \equiv$ the first best effort choice.

- ▶ A debt contract, by achieving maximal effort among available monotonic payoff functions, enables the entrepreneur to get closer to the 'first best.'