

Lecture XXI: Estimation of Endogenous Debt

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Readings for next time

Risk Endogenous Model of Aggregate Farm Debt

Empirical Estimation with Dairy Data

Kansas Farm Data

ARMS Data

Readings for Wednesday

- ▶ Darrough, M.N. and N.M. Stoughton. 1986. Moral Hazard and Adverse Selection: The Question of Financial Structure. *Journal of Finance* 41(2), 501-513.
- ▶ Innes, R.D. 2002. Market Failures and Second-Best Analysis with a Focus on Nutrition, Credit, and Incomplete Markets. *Handbook on Agricultural Economics*. Elsevier, pp. 1828-1892.
- ▶ Innes, R.D. 1990. Limited Liability and Incentive Contracting with Ex-Ante Action Choices. *Journal of Economic Theory* 52(1), 45-67.
- ▶ Stiglitz, J.E. and A. Weiss. 1981. Credit Rationing in Markets with Imperfect Information. *American Economic Review* 71(3), 393-410.

Risk Endogenous Model of Aggregate Farm Debt

- ▶ Moss, C.B., J.S. Shonkwiler, and S.A. Ford. 1990. A Risk Endogenous Model of Aggregate Agricultural Debt. *Agricultural Finance Review* 56, 85-97.
- ▶ Starting with the basic Collins result

$$\delta^* = 1 - \frac{\rho\sigma_A^2}{[\mu_A - K]}. \quad (1)$$

- ▶ The change in debt-to-asset level is then written as

$$d\delta^* = -\frac{\rho}{[\mu_A - K]}d\sigma_A^2 + \frac{\rho\sigma_A^2}{[\mu_A - K]^2}[d\mu_A - dK] \quad (2)$$

- ▶ Reformulating this model to focus on the change in the level of debt

$$D = A - \frac{A\rho\sigma_A^2}{[\mu_A - K]}. \quad (3)$$

- ▶ Differentiating Equation 3

$$dD = \left[1 - \frac{\rho\sigma_A^2}{\mu_A - K} \right] dA + \left[-\frac{\rho}{\mu_A - K} d\sigma_A^2 + \frac{\rho\sigma_A^2}{[\mu_A - K]^2} d\mu_A - \frac{\rho\sigma_A^2}{[\mu_A - K]^2} dK \right] A \quad (4)$$

- The basic concept is that we could derive a linear form

$$D_t = f(A_t, R_{At}, \sigma_{At}^2, r_t) \Rightarrow D_t = a_0 + a_1 A_t + a_2 R_{At} + a_3 \sigma_{At}^2 + a_4 r_t \quad (5)$$

Basically changes in the level of debt are explained by changes in the level of assets, changes in the rate of return on assets, changes in the variance of the rate of returns on assets, and changes in the interest rate.

- Following the standard approach at time, we difference the model to allow for partial adjustment

$$\Delta d_t = \alpha_0 + \alpha_1 \Delta d_{t-1} + \alpha_2 \Delta r_t + \alpha_3 R_t^* + \alpha_4 V_t^* \quad (6)$$

where d is the real level of debt, r_t is the real interest rate, R_t^* is the expected central moment of the real rate of return on assets

$$R_t^* = \beta(L) R_t + \xi_t \Rightarrow R_t^* = \beta_0 + \beta_1 R_{t-1} + \beta_2 R_{t-2} + \beta_3 R_{t-3} + \xi_t. \quad (7)$$

- The error term is then modeled as a restricted form of an Autoregressive Conditional Heteroskasticity formulation

$$h_t = \gamma_0 + \gamma_1 (0.3 |\xi_{t-1}| + 0.2 |\xi_{t-2}| + 0.1 |\xi_{t-3}|) \quad (8)$$

- ▶ Putting the model together

$$\begin{aligned} \max_{\theta} L(\theta) = & - \frac{[\Delta d_t - \alpha_0 - \alpha_1 \Delta d_{t-1} - \alpha_2 \Delta r_t + \alpha_3 (\beta_0 + \beta_1 R_{t-1} + \beta_2 R_{t-2} + \beta_3 R_{t-3}) + \alpha_r h_t]^2}{\omega_{11}} \\ & - \frac{\xi_t^2}{h_t} \\ \xi_t = & R_t - \beta_0 - \beta_1 R_{t-1} - \beta_2 R_{t-2} + \beta_3 R_{t-3} \\ h_t = & \gamma_0 + \gamma_1 (0.3 |\xi_{t-1}| + 0.2 |\xi_{t-2}| + 0.1 |\xi_{t-3}|) \end{aligned} \quad (9)$$

Generalized Least Squares Estimates

Variable	Parameter	Standard Deviation	<i>t</i> -Value
Constant	0.003	0.012	0.252
Δd_{t-1}	0.683	0.130	5.27
Δr_t	-0.020	0.005	-4.15
R_t^*	1.126	0.277	4.06
V_t^*	-0.251	0.073	-3.46

Empirical Estimation with Dairy Data

- ▶ Ahrendsen, B.L., R.N. Collender, and B.L. Dixon. 1994. An Empirical Analysis of Optimal Farm Capital Structure Decisions. *Agricultural Finance Review* 54, 108-119.
- ▶ This paper builds of the basic Moss, Ford and Boggess model – incorporating values from different tax shields. In addition, the study uses farm-level data (the North Carolina Dairy Farm Business Summary for 110 dairy farms).

$$\begin{aligned}\bar{r}_W &= \frac{(\tilde{r}_A - r_D\delta)}{(1 - \delta)} \\ &= \frac{(\tilde{r}_G + \tilde{r}_O - r_D\delta)}{(1 - \delta)} \\ V(\tilde{r}_W) &= \frac{\sigma_A^2}{(1 - \delta)^2}\end{aligned}\tag{10}$$

- ▶ Building on the concept of capital gains exclusion, Ahrendsen et al. introduce terms pA where p is the depreciation rate (pA is then amount of asset written off in a given year) and $p_{ITC}A$ where p_{ITC} is the amount of Investment Tax Credit received.
- ▶ The change in wealth can then be depicted as

$$\tilde{W}_1 = W_G + \tilde{r}_G A [1 - \psi\tau] + [\tilde{r}_O A - r_D D - pA] [1 - \tau] + pA + p_{ITC} A_n$$

$$E [\tilde{W}_1] = W_0 + \mu_G A [1 - \psi\tau] + [\mu_0 A - r_D D - pA] [1 - \tau] + pA + p_{ITC} A_n$$

$$V [\tilde{W}_1] = A^2 \left[(1 - \psi\tau)^2 \sigma_G^2 + (1 - \tau)^2 \sigma_O^2 + 2(1 - \psi\tau)(1 - \tau) \sigma_{OG} \right] = A^2 \sigma_A^2 \quad (11)$$

- The authors then propose estimating the certainty equivalent of wealth instead of the certainty equivalent of the rate of return

$$CE \left(\tilde{W}_1(A) \right) = E \left(\tilde{W}_1 \right) - \frac{\gamma}{2} V \left(\tilde{W}_1 \right)$$
$$\Rightarrow A^* = \frac{\mu_G (1 - \psi\tau) + (\mu - r_G) (1 - \tau) + \tau p + p_{ITC}}{\gamma \sigma_A^2} \quad (12)$$

- ▶ The empirical specification is then based on the numerator from Equation 12.
- ▶ First, the authors hypothesize the base model – instantaneous adjustment

$$A_{it}^* = \frac{(SPREAD_{it}^{\beta_1})}{\gamma \sigma_{A,it}^{2\beta_2}} + u_{it} \quad (13)$$

with the $\beta_1 = 1$ and $\beta_2 = 1$ representing the standard Collins model result.

- ▶ As an alternative model, the authors suggest a partial adjustment model

$$A_{it} - A_{i,t-1} = \phi [A_{it}^* - A_{i,t-1}]$$

$$A_{it} = \phi \frac{SPREAD_{it}^{\beta_1}}{\gamma \sigma_{A,it}^{2\beta_2}} + (1 - \phi) A_{i,t-1} + e_{it} \quad (14)$$

- ▶ The real issue in any of these models is how to measure the business risk – the variance of the rate of returns on agricultural assets.
- ▶ "An estimate of farmer i 's business risk, σ_A^2 , is the variance of the sum of after-tax operating rate of return on assets and rate of capital gain. This business-risk estimate measures how susceptible a farm is to weather and other sources of variability beyond the control of the farmer. Also, this measure of business risk is consistent with measures used in the corporate finance literature."

Kansas Farm Data

- ▶ Jensen, F.E. and L.N. Langemeier. 1996. Optimal Leverage with Risk Aversion: Empirical Evidence. *Agricultural Finance Review* 56, 85-97.
- ▶ This paper follows the concept of building additional considerations or numerating the factors that determine expected returns to assets and the variance of the return to assets.
- ▶ Starting with a slightly different base model

$$\tilde{R}_A = \frac{\tilde{\pi}}{L + \theta} \quad (15)$$

where $\tilde{\pi}$ is the operating income before depreciation, interest, and taxes, L is the average land value, and θ is the average value of all other assets.

- ▶ This development leads to the standard formulation for the rate of return on equity

$$\tilde{R}_E = \left\{ \frac{\tilde{\pi}}{L + \theta} - r\delta \right\} \left\{ \frac{1}{1 - \delta} \right\} \quad (16)$$

- ▶ The authors then expand the model to include:
 - ▶ State and federal income taxes
 - ▶ Social security
 - ▶ Long-term capital gains
 - ▶ Depreciation shields
 - ▶ Returns to operator's unpaid labor and management

$$\tilde{R}_E = \left\{ \frac{\tilde{\pi}(1-T) - F + \tilde{r}_g(L+\theta)(1-\Psi T) + \gamma\Gamma + \Delta\theta T}{L+\theta} - [r\delta](1-T) \right\} \cdot \left\{ \frac{1}{1-\delta} \right\} \quad (17)$$

- ▶ Apart from the variables that we have been working with
 - ▶ γ is the assets purchased which are subject to depreciation
 - ▶ Δ is the total accounting depreciation as a percentage of θ
 - ▶ Γ is the investment tax credit as a percentage of γ
 - ▶ F is the returns to owner's management and laobr

- Following a now standard approach, the authors derive the expected rate of return and the variance of the rate of return on equity

$$\bar{R}_E = \left\{ \frac{\bar{\pi}(1-T) + F + \bar{r}_g(L+\theta)(1-\psi T) + \gamma\Gamma + \Delta\theta T}{L+\theta} - [r\delta](1-T) \right\} \cdot \left\{ \frac{1}{1-\delta} \right\} \quad (18)$$

$$\sigma_E^2 = \left\{ \frac{1}{(1-\delta)^2(L+\theta)^2} \right\} \cdot \left\{ (1-T)^2 \sigma_\pi^2 + 2(1-T)(L+\theta)(1-\psi T) \sigma_{\pi r_g} + (1-\psi T)^2(L+\theta)^2 \sigma_{r_g}^2 \right\} \quad (19)$$

$$\delta^* = 1 - \rho \frac{(1-T)^2 \sigma_\pi^2 + 2(1-T)(L+\theta)(1-\Psi T) \sigma_{\pi r_g} + (1-\Psi T)^2 (L+L\theta)^2 \sigma_{r_g}^2}{(L+\theta) \{ \bar{\pi}(1-T) - F\bar{r}_g(L+\theta)(1-\Psi T) + \gamma\Gamma + \Delta\theta T + r(T-1)(L+\theta) \}} \quad (20)$$

- Following the standard approach (i.e., the dynamic model from Moss, Shonkwiler and Ford), the paper posits a general functional form

$$\delta^* = f \left[\bar{\pi}, \bar{r}_g, L, \theta, \rho, \sigma_\pi^2, \sigma_{r_g}^2, \sigma_{\pi r_g}, T, \gamma\Gamma, \Psi, \Delta, F, r \right] \quad (21)$$

- The first-order approximation then becomes

$$\delta_{jt}^* = \beta_0 + \beta_1 \bar{\pi}_{jt} + \beta_2 \bar{r}_{gjt} + \beta_3 L_{jt} + \beta_4 \theta_{jt} + \beta_5 \sigma_{\pi t}^2 + \beta_6 \sigma_{r_g t}^2 + \beta_7 \sigma_{\pi r_g t} + \beta_8 T_{jt} + \beta_9 \gamma \Gamma_{jt} + \beta_{10} \Psi_{jt} + \beta_{11} \Delta_{jt} + \beta_{12} F_{jt} + \beta_{13} r_t + e_{jt} \quad (22)$$

- Finally building on Equation 22 the model is reformulated as a partial adjustment model

$$\delta_{jt} - \delta_{j,t-1} = \alpha [\delta_{jt}^* - \delta_{j,t-1}]$$

$$\delta_{jt} = b_0 + b_1 \bar{\pi}_{jt} + b_2 \bar{r}_{gjt} + b_3 L_{jt} + b_4 \theta_{jt} + b_5 \sigma_{\pi t}^2 + b_6 \sigma_{r_g t}^2 + b_7 \sigma_{\pi r_g t} + b_8 T_{jt} + b_9 \gamma \Gamma_{jt} + b_{10} \Psi_{jt} + b_{11} \Delta_{jt} + b_{12} F_{jt} + b_{13} r_t + b_{14} \delta_{j,t-1} + e_{jt} \quad (23)$$

ARMS Data

- ▶ Moss, C.B. and A.K. Mishra. 2013. Debt-Equity Structure and Risk Balancing by Farm Businesses: An Application with ARMS Data. Food and Resource Economics Department, University of Florida Working Paper.

- ▶ Taking the first-order Taylor series approximation of the natural logarithm of both sides, Equation the standard Collins's model yields

$$\begin{aligned} \ln(\delta^*) &= \ln(\delta_0) + \frac{1}{1 - \frac{\rho\sigma_0^2}{x_0}} \left[-\frac{\rho}{x_0} (\sigma^2 - \sigma_0^2) \right] + \frac{1}{1 - \frac{\rho\sigma_0^2}{x_0}} \left[\frac{\rho\sigma_0^2}{x_0^2} (x - x_0) \right] \\ &= \ln(\delta_0) - \frac{\rho}{x_0 - \rho\sigma_0^2} (\sigma^2 - \sigma_0^2) + \frac{\rho\sigma_0^2}{x_0 - \rho\sigma_0^2} \frac{1}{x_0} (x - x_0) \end{aligned} \quad (24)$$

where $x = \mu_A - K$ or the expected returns over the cost of capital for a particular region or time period, and x_0 and σ_0^2 are the sample means used as the point of approximation.

- Next, by logarithmic differentiation

$$\begin{aligned}\sigma^2 - \sigma_0^2 &= d\sigma^2 = \sigma^2 d \ln(\sigma^2) \Leftarrow d \ln(\sigma^2) = \frac{1}{\sigma^2} d \ln(\sigma^2) \\ &\approx \sigma_0^2 (\ln(\sigma^2) - \ln(\sigma_0^2))\end{aligned}\tag{25}$$

$$\begin{aligned}x - x_0 &= dx = x d \ln(x) \Leftarrow d \ln(x) = \frac{1}{x} d \ln(x) \\ &\approx x_0 (\ln(x) - \ln(x_0))\end{aligned}$$

- Substituting these results into Equation 24 yields

$$\begin{aligned}\ln(\delta^*) &= \ln(\delta_0) - \frac{\rho}{x_0 - \rho\sigma_0^2} \sigma_0^2 (\ln(\sigma^2) - \ln(\sigma_0^2)) + \frac{\rho\sigma_0^2}{x_0 - \rho\sigma_0^2} \frac{1}{x_0} x_0 (\ln(x) - \ln(x_0)) \\ &= \ln(\delta_0) - \frac{\rho\sigma_0^2}{x_0 - \rho\sigma_0^2} (\ln(\sigma^2) - \ln(\sigma_0^2)) + \frac{\rho\sigma_0^2}{x_0 - \rho\sigma_0^2} (\ln(x) - \ln(x_0)) \\ &= \left[\ln(\delta_0) + \frac{\rho\sigma_0^2}{x_0 - \rho\sigma_0^2} \ln(\sigma_0^2) - \frac{\rho\sigma_0^2}{x_0 - \rho\sigma_0^2} \ln(x_0) \right] - \frac{\rho\sigma_0^2}{x_0 - \rho\sigma_0^2} \ln(\sigma^2) + \frac{\rho\sigma_0^2}{x_0 - \rho\sigma_0^2} \ln(x)\end{aligned}\tag{26}$$

- Transforming the model in Equation 26 into an estimatable form

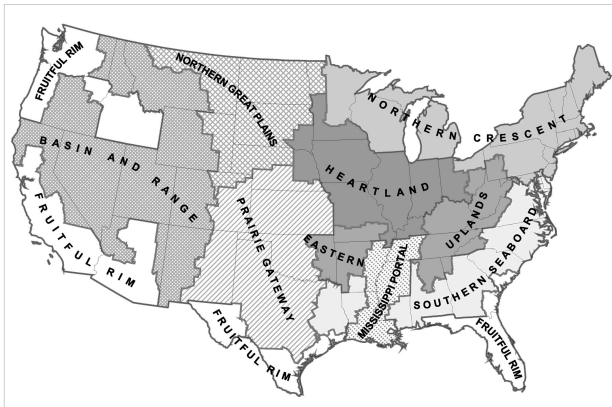
$$\begin{aligned} \ln(\delta^*) &= \beta_0 + \beta_1 \ln(\sigma^2) + \beta_2 \ln(x) \\ \beta_0 &= \left[\ln\left(1 + \frac{\rho\sigma_0^2}{x_0}\right) + \frac{\rho\sigma_0^2}{x_0 - \rho\sigma_0^2} \ln(\sigma_0^2) - \frac{\rho\sigma_0^2}{x_0 + \rho\sigma_0^2} \ln(x_0) \right] \\ \beta_1 &= -\frac{x_0}{x_0 - \rho\sigma_0^2} \\ \beta_2 &= \frac{x_0}{x_0 - \rho\sigma_0^2} \end{aligned} \quad (27)$$

or $\beta_1 = -\beta_2$.

Empirical Model

$$\ln(DA_{it}) = \beta_0 + \beta_1 \ln(\pi_{it}) + \beta_2 \ln(VROA_{it}) \quad (28)$$
$$i = 1, \dots, 9 \text{ and } t = 1996, \dots, 2003$$

Regions



Debt Structure and Risk by Farming Regions in the U.S.

Variables	Parameter Estimates
Intercept	-2.988 (0.349)
Log of Margin (difference between the average rate of return on assets and the average cost of capital)	0.163** (0.083)
Log of Variance of Returns on Assets (including capital gains)	-0.081*** (0.023)
R-Squared	0.66
Fixed Effects Year	1.55
Region	2.72**