

Lecture XIX: Optimal Debt – The Risk Balancing Model

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Readings for next time

Introduction to the Risk-Balancing Model

Expected Utility and Risk Balancing

Readings for next time

- ▶ Featherstone, A.M., C.B. Moss, T.G. Baker, and P.V. Preckel. 1988. The Theoretical Effects of Farm Policies on Optimal Leverage and the Probability of Equity Loss. *American Journal of Agricultural Economics* 70(3), 572-579.
- ▶ Moss, C.B., S.A. Ford, and W.G. Boggess. 1989. Capital Gains, Optimal Leverage, and the Probability of Equity Loss: A Theoretical Model. *Agricultural Finance Review* 49, 1927-34.
- ▶ Ramirez, O.A., C.B. Moss, and W.G. Boggess. 1997. A Stochastic Optimal Control Formulation of the Consumption/Debt Decision. *Agricultural Finance Review* 47, 29-39.

After That

- ▶ Darrough, M.N. and N.M. Stoughton. 1986. Moral Hazard and Adverse Selection: The Question of Financial Structure. *Journal of Finance* 41(2), 501-513.
- ▶ Innes, R.D. 2002. Market Failures and Second-Best Analysis with a Focus on Nutrition, Credit, and Incomplete Markets. *Handbook on Agricultural Economics*. Elsevier, pp. 1828-1892.
- ▶ Innes, R.D. 1990. Limited Liability and Incentive Contracting with Ex-Ante Action Choices. *Journal of Economic Theory* 52(1), 45-67.
- ▶ Stiglitz, J.E. and A. Weiss. 1981. Credit Rationing in Markets with Imperfect Information. *American Economic Review* 71(3), 393-410.

Introduction to the Risk-Balancing Model

- ▶ Gabriel, S.C. and C.B. Baker. 1980. Concepts of Business and Financial Risk. *American Journal of Agricultural Economics* 62(3), 560-564.
- ▶ The big-picture of this article is that there are two types of risk experienced by the firm:
- ▶ The first type of risk is the business risk – the normal risk associated with random prices and yields

$$p'y - w'x \ni: p \sim N(\mu_p, \Sigma_p) \ y \sim N(\mu_y, \Sigma_y). \quad (1)$$

- ▶ The second source of risk is the financial risk – the risk financial decisions such as the debt choice.
- ▶ "The financial risk typically leverages the business risk – expands it exponentially.
- ▶ We suggest that there is also a financial response to business risk modification. The differentiation is important in that business risk and financial risk may well be trade-offs in the risk behavior of farmers. Thus, a decline in business risk would lead to the acceptance of greater financial risk, reducing the effects of the diminished business risk on total risk."

- ▶ "Business Risk is defined to be the risk inherent in the firm independent of the way it is financed."
- ▶ "... financial risk is defined to be the added variability of the net cash flows of the owners equity that results from the fixed financial obligation associated with debt financing and cash leasing."

- ▶ Gabriel and Baker develop the financial risk as

$$FR = \frac{\sigma_2}{\bar{c}'_x - I} - \frac{\sigma_1}{\bar{c}'_x} \quad (2)$$

- ▶ σ_1 is the standard deviation of the operating cash flows (i.e., the cash flows of the farm without debt or leasing obligations) and σ_2 is the standard deviation of the returns with the debt or leasing obligations.
- ▶ $\bar{c}'_x$ is the netput profit function – $\bar{c}$ is the mean set of prices and x is positive for a farm output and negative for an input.
- ▶ I is the fixed level of debt obligation.

- ▶ We are going to develop a little different model of the interaction between business and financial risk with the Collins model – however, Gabriel and Baker reformulate Equation 2 to focus on the component of financial risk

$$FR = \frac{\sigma_2 - \sigma_1}{\bar{c}'_X - I} + \frac{\sigma_1}{\bar{c}'_X} \frac{I}{\bar{c}'_X - I} \quad (3)$$

- ▶ "Consider the case where leverage does not induce a change in the variability of net cash flows (i.e., where $\sigma_1 = \sigma_2$). When $\sigma_1 = \sigma_2$, the σ_1 first term of the right side of Equation [3] becomes zero, leaving Equation [4]

$$FR = \frac{\sigma_1}{\bar{c}'_X} \frac{I}{\bar{c}'_X - I}. \quad (4)$$

We see that financial risk is determined by the degree of business risk inherent in the firm $\sigma_1/(\bar{c}'_X)$, and the relation $I/(\bar{c}'_X - I)$ which is determined by the financing decision."

- ▶ As an interesting point, Gabriel and Baker consider the scenario where the change in leverage implies a change in relative risk – $\sigma_2 < \sigma_1$. This concept may be lost in the other studies.

- ▶ "Risk balancing refers to the adjustment in the components of total risk (i.e., business and financial risk) that results from an exogenous shock to the existing balance. This shock is to be represented by a change in business risk and is viewed within the context of a total risk constraint."
- ▶ Assuming no leverage-induced change in business risk ($\sigma_1 = \sigma_2$)

$$TR = \frac{\sigma_1}{\bar{c}'_X - I}, \text{ or} \quad (5)$$

$$TR = \frac{\sigma_1}{\bar{c}'_X} \frac{\bar{c}'_X}{\bar{c}'_X - I}$$

- ▶ Gabriel and Baker then formulate a risk constraint (we will quickly replace this constraint with a utility formulation)

$$\frac{\sigma_1}{\bar{c}'_X} \frac{\bar{c}'_X}{\bar{c}'_X - I} \leq \beta \quad (6)$$

- ▶ Suppose that, due to a change in an exogenous source such as price support policy or foreign grade policy, σ_1 declines, leaving all the other variables unchanged.
- ▶ Total Risk will have declined proportionally, leaving slack in the risk constraint specified in Equation [6].
- ▶ Financial risk also will have fallen proportionally.

- ▶ Because financial risk contributes to the total risk of the firm and is a function of the financing decision, internal capital rationing may occur in order to comply with the risk constraint.
- ▶ Therefore, one might expect debt use and, consequently, financial risk, to adjust to the changes in the level of business risk.

- ▶ Finally, Gabriel and Baker introduce the concept of liquidity and the probability of meeting cash-flow requirements.
- ▶ As a first step, consider the probability of meeting the cash flow requirements of debt and family living expenses as

$$P(c'x + \mu - I \leq z) = \alpha \quad (7)$$

$$P(c'x + \mu - I \leq z) \leq \alpha$$

where $c'x$ is now a random outcome, μ is a liquidity reserve, and α is a given probability level.

- ▶ For more discussion of the liquidity nature of the debt decision read Moss pp. 171–190.

Expected Utility and Risk Balancing

- ▶ Collins, R.A. 1985. Expected Utility, Debt-Equity Structure, and Risk Balancing. *American Journal of Agricultural Economics* 67(3), 627-629.
- ▶ The goal here is to replace Gabriel and Baker's ad hoc risk balancing constraint in Equation 6 with a theoretic specification – utility.
- ▶ As the first part of this process, I need to briefly talk about the DuPont Expansion (Moss pp. 96-97 – we will expand on this analysis more later).

- ▶ One very general form of the DuPont expansion is

$$\frac{R_E}{E} = \frac{S - C}{S} \cdot \frac{S}{A} \cdot \frac{A}{E} \quad (8)$$

- ▶ R_E/E is the rate of return to equity.
- ▶ $(S - C)/S$ is the gross margin – S is the total value of sales and C is the associated cost.
- ▶ S/A is referred to as the asset efficiency – A is the firm's total asset level.
- ▶ A/E is the leverage ratio

- Collins changes this basic expression to

$$\frac{r_P}{E} \equiv \frac{r_P}{A} \cdot \frac{A}{E} \quad (9)$$

where r_P is the "expected return to the portfolio of enterprises." Substituting $\delta = D/A$

$$\frac{r_P}{E} \equiv \frac{r_P}{A} \cdot \frac{1}{1 - \delta}. \quad (10)$$

- Finally, Collins introduces the cost of debt capital as $-\delta K$ yielding

$$R_E = \left[\frac{r_P}{A} + i - \delta K \right] \cdot \frac{1}{1 - \delta} = [R_A - \delta K] \cdot \frac{1}{1 - \delta} \quad (11)$$

where i is the rate of capital appreciation (change in land value), R_E is the rate of return to equity, and R_A is the rate of return to assets.

- ▶ If we assume that the rate of return on assets is normally distributed $R_A \sim N(\mu_A, \sigma_A^2)$ then the rate of return to equity is distributed normally

$$R_E \sim N\left(\frac{\mu_A - \delta K}{1 - \delta}, \frac{\sigma_A^2}{(1 - \delta)^2}\right) \quad (12)$$

- ▶ If we assume a negative exponential utility function

$$U(W) = -\exp(-\rho W) \quad (13)$$

given the assumptions about normality, we can rewrite the utility function as

$$V(\delta) = [\mu_A - \delta K] \frac{1}{1 - \delta} - \frac{\rho}{2} \sigma_A^2 \left(\frac{1}{1 - \delta}\right)^2. \quad (14)$$

- ▶ Taking the derivative of Equation 14 with respect to δ and solving for the zero yields

$$\delta^* = 1 - \frac{\rho\sigma_A^2}{\mu_A - K} \ni: -\frac{\rho}{2}\sigma_A^2 < 0. \quad (15)$$

- ▶ This derivation adds some additional rigor to Gabriel and Baker – Note the risk balancing implications

$$\frac{\partial\delta^*}{\partial\sigma_A^2} = -\frac{\rho}{\mu_A - K} \ll 0 \ni: \rho > 0 \text{ and } \mu_A - K > 0. \quad (16)$$

► Also note that

$$\frac{\partial \delta^*}{\partial \mu_A} = \frac{\rho \sigma_A^2}{(\mu_A - K)^2} \gg 0 \quad (17)$$

$$\frac{\partial \delta^*}{\partial K} = -\frac{\rho \sigma_A^2}{(\mu_A - K)^2} \ll 0 \quad \mu_A > K.$$