

Lecture I: Neoclassical Market Model for Capital I

Charles B. Moss

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Completing the Demand Model

Extending to Production

Robinson Crusoe Without Trade

Robinson Crusoe With Trade

Completing the Demand Model

- ▶ Recalling the solution of the consumer's problem from Lecture 1

$$\begin{aligned} &\text{Max } q_1^{0.30} q_2^{0.60} \\ &\text{s.t. } q_1 + \phi q_2 \leq q_1^* + \phi q_2^* \end{aligned}$$

$$L = q_1^{0.30} q_2^{0.60} + \lambda [15 + 20\phi - q_1 - \phi q_2] \quad (1)$$

$$\Rightarrow \begin{cases} q_1 = \frac{15 + 20\phi}{3} \\ q_2 = \frac{30 + 40\phi}{3\phi} \end{cases} .$$

- Next, assume that a second consumer has a slightly different utility function, but the same endowment

$$\begin{aligned} &\text{Max } q_1^{0.45} q_2^{0.45} \\ &\text{s.t. } q_1 + \phi q_2 \leq 15 + \phi 20 \end{aligned}$$

$$L = q_1^{0.45} q_2^{0.45} - \lambda [15 + \phi 20 - q_1 - \phi q_2]$$

$$\left. \begin{aligned} \frac{\partial L}{\partial q_1} &= 0.45 \frac{U}{q_1} - \lambda = 0 \\ \frac{\partial L}{\partial q_2} &= 0.45 \frac{U}{q_2} - \phi \lambda = 0 \end{aligned} \right\} \Rightarrow \frac{0.45 \frac{U}{q_1}}{0.45 \frac{U}{q_2}} = \frac{1}{\phi} \Rightarrow \frac{q_2}{q_1} = \frac{1}{\phi} \Rightarrow q_2 = \frac{q_1}{\phi}. \quad (2)$$

- Substituting this result into the budget constraint

$$15 + \phi 20 - q_1 - \phi \frac{q_1}{\phi} = 0$$

$$2q_1 = 15 + \phi 20$$

$$q_1 = \frac{15 + 20\phi}{2} \quad (3)$$

$$q_2 = \frac{15 + 20\phi}{2\phi}.$$

- First, notice that if $\phi = 1.0$ then $q_1 = 17.5$ and $q_2 = 17.5$. Further, the value of the endowment is 35. If $\phi = 1.5$ then $q_1 = 22.5$ and $q_2 = 15$. The value of the endowment becomes 45.

Consumption Market Equilibrium

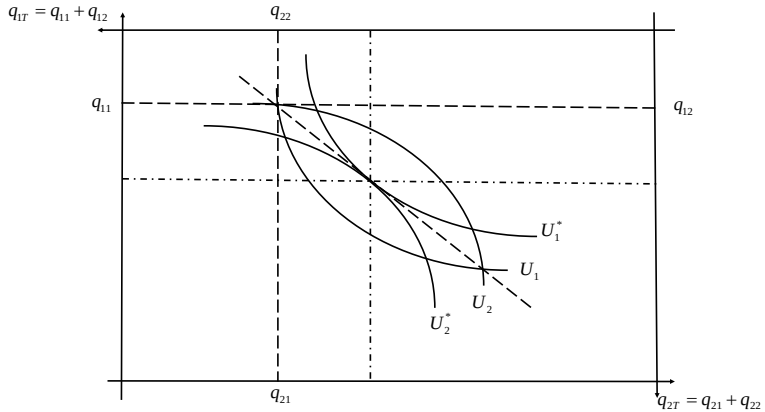
Good	0.75		1.00		1.50	
	Consumption	Sell	Consumption	Sell	Consumption	Sell
<i>Consumer 1</i>						
q_{11}	10.0	5.0	11.7	3.3	15.0	0.0
q_{21}	26.7	-6.7	23.3	-3.3	20.0	0.0
<i>Consumer 2</i>						
q_{12}	15.0	0.0	17.5	-2.5	22.5	-7.5
q_{22}	20.0	0.0	17.5	2.5	15.0	5.0
<i>Net Market</i>						
q_1		5.0		0.8		-7.5
q_2		-6.7		-0.8		5.0

- $$\begin{aligned} q_{11} \left\{ = \frac{15 + 20\phi}{3} \right\} + q_{12} \left\{ = \frac{15 + 20\phi}{2} \right\} &= 2 \times 15 \\ q_{21} \left\{ = \frac{30 + 40\phi}{3\phi} \right\} + q_{22} \left\{ = \frac{15 + 20\phi}{2\phi} \right\} &= 2 \times 20 \end{aligned} \quad (4)$$

- $$\left(\frac{15+20\phi}{3} + \frac{15+20\phi}{2} - 30\right)^2 + \left(\frac{30+40\phi}{3\phi} + \frac{15+20\phi}{2\phi} - 40\right)^2 \quad (5)$$

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Edgeworth Box



Extending to Production

- ▶ Following Moss pp. 16–20, we start by developing the Robinson Crusoe economy.
- ▶ The stranded economist's utility is a function of consumption this year and next year

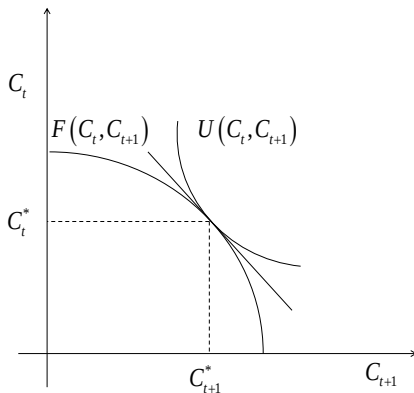
$$U(C_t, C_{t+1}) = C_t^{0.60} C_{t+1}^{0.40} \quad (6)$$

- ▶ Instead of a linear budget constraint, we hypothesize that Robinson's consumption in each period is constrained by a production possibility frontier

$$10000 \geq C_t^2 + \frac{C_{t+1}^2}{4} \quad (7)$$

(Basically the production possibility frontier is the points on an ellipse in the space where both consumptions are positive).

Robinson Crusoe Capital Market



- Working out the simple mathematics – Crusoe's problem becomes

$$\begin{aligned} \max_{C_t, C_{t+1}} U(C_t, C_{t+1}) &= C_t^{0.6} C_{t+1}^{0.4} \\ \text{s.t. } 10000 &\geq C_t^2 + \frac{C_{t+1}^2}{4}. \end{aligned} \quad (8)$$

- $$L = C_t^{0.6} C_{t+1}^{0.4} + \lambda \left(10000 - C_t^2 - \frac{C_{t+1}^2}{4} \right)$$

$$\frac{\partial L}{\partial C_{t+1}} = 0.4 \frac{U(.)}{C_{t+1}} - \lambda \frac{C_{t+1}}{2} = 0$$

$$\frac{\partial L}{\partial \lambda} = 10000 - C_t^2 - \frac{C_{t+1}^2}{4} = 0.$$

- ▶ Following our solution mechanism for Cobb-Douglas functions

$$\frac{\frac{0.6U(.)}{C_t}}{\frac{0.4U(.)}{C_{t+1}}} = \frac{\lambda 2C_t}{\lambda \frac{C_{t+1}}{2}} \Rightarrow C_{t+1}^2 = \frac{8}{3}C_t^2. \quad (10)$$

Substituting Equation 10 into the nonlinear constraint

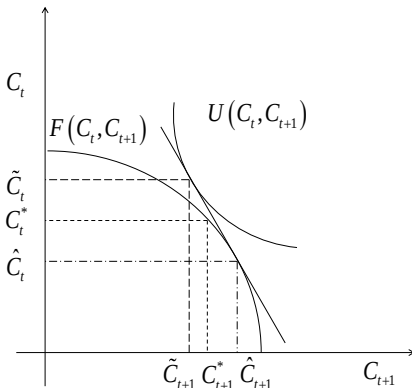
$$10000 - C_t^2 - \frac{1}{4} \left(\frac{8}{3}C_t^2 \right) = 0 \Rightarrow C_t = 77.460. \quad (11)$$

- $$MRS = \frac{\partial C_t}{\partial C_{t+1}} = \frac{\frac{\partial U(.)}{\partial C_{t+1}}}{\frac{\partial U(.)}{\partial C_t}} = -\frac{2}{3} \frac{C_t}{C_{t+1}} = -\frac{2}{3} \frac{77.460}{126.492} = -0.4082. \quad (12)$$

Robinson Crusoe with Trade

- ▶ Next, we consider the possibility of trade. As depicted in Figure 17, Crusoe consumes $\tilde{C}_t$ of which $\hat{C}_t$ is from his original endowment. The difference $\tilde{C}_t - \hat{C}_t$ is "borrowed" from the savages on the other island. In the second period Crusoe produces $\hat{C}_{t+1}$ potatoes while consuming $\tilde{C}_{t+1}$ potatoes. Thus, he pays $\tilde{C}_{t+1} - \hat{C}_{t+1}$ potatoes back to the savages.

Robinson Crusoe Capital Market



- ▶ The implicit price ratio behind the trade is determined by the "excess supply" relationship derived implicitly in Equation 3. Specifically, the first consumer's excess supply of good 1 q_1^e can be written as

$$q_1^e = 15 - \frac{15 + 20\phi}{2}. \quad (13)$$

In the current scenario, the excess supply of current consumption would be stated

$$\tilde{C}_t - \hat{C}_t = f^*(\phi). \quad (14)$$

To make this problem somewhat more tractable, it is useful to break this problem up into two parts.

- First, we ask for a given interest rate r what combination of production in the current period $\hat{C}_t$ and production next period $\hat{C}_{t+1}$ will Crusoe supply to the market

$$\text{Max } C_t + \frac{1}{1+r} C_{t+1} \quad (15)$$

$$\text{s.t. } 10000 \geq C_t^2 + \frac{C_{t+1}^2}{4}.$$

If we assume an interest rate of 8 percent
 $1/(1+r) = 0.925925$

$$\text{Max } C_t + 0.925925 C_{t+1} \quad (16)$$

$$\text{s.t. } 10000 \geq C_t^2 + \frac{C_{t+1}^2}{4}.$$

- In Lagrangian form

$$L = C_t + 0.925925C_{t+1} + \lambda \left(10000 - C_t^2 - \frac{C_{t+1}^2}{4} \right)$$

$$\frac{\partial L}{\partial C_t} = 1 - 2C_t\lambda = 0 \quad (17)$$

$$\frac{\partial L}{\partial C_{t+1}} = 0.925925 - \frac{2C_{t+1}}{4}\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 10000 - C_t^2 - \frac{C_{t+1}^2}{4} = 0.$$

- Solving the first two first-order conditions yields $C_{t+1} = 3.7037C_t$. Substituting this result into the constraint yields

- ▶ Substituting this result into the previous result yields

$$\begin{aligned} y &= (3.7037x) = 175.981 \\ \lambda &= \frac{1}{2x} = 0.01052. \end{aligned} \tag{19}$$

- ▶ The second question is then what combination of consumption today versus tomorrow is demand by Crusoe. To accomplish, this we start by computing Crusoe's implicit budget constraint

$$Y = 47.515 + 0.925925 (175.981) = 210.460. \tag{20}$$

- $$\begin{aligned} & \text{Max } C_t^{0.6} C_{t+1}^{0.4} \\ \text{s.t. } & 210.460 \geq C_t + 0.925925 C_{t+1} \end{aligned} \quad (21)$$