

Lecture VIII: Grossman and Hart – Vertical and Lateral Integration [Part II]

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September 24, 2013

I. Nonintegrated Decisions

- A. We have developed a model where each manager $i \in \{1, 2\}$ maximize

$$B_i [a_i, \phi_i (q_1, q_2)] \quad (1)$$

- B. where the the gains from the renegotiation.

1. The benefit of these various strategies for each individual then becomes

$$\begin{aligned} \phi_1 (\{ q_{11} \cdot q_{21} \}) &= \begin{pmatrix} \phi_1 (q_{11,1}, q_{21,1}) & \phi_1 (q_{11,2}, q_{21,1}) & \cdots & \phi_1 (q_{11,k_1}, q_{21,1}) \\ \phi_1 (q_{11,1}, q_{21,2}) & \phi_1 (q_{11,2}, q_{21,2}) & \cdots & \phi_1 (q_{11,k_1}, q_{21,2}) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1 (q_{11,1}, q_{21,k_2}) & \phi_1 (q_{11,2}, q_{21,k_2}) & \cdots & \phi_1 (q_{11,k_1}, q_{21,k_2}) \end{pmatrix} \\ \phi_2 (\{ q_{11} \cdot q_{21} \}) &= \begin{pmatrix} \phi_2 (q_{11,1}, q_{21,1}) & \phi_2 (q_{11,2}, q_{21,1}) & \cdots & \phi_2 (q_{11,k_1}, q_{21,1}) \\ \phi_2 (q_{11,1}, q_{21,2}) & \phi_2 (q_{11,2}, q_{21,2}) & \cdots & \phi_2 (q_{11,k_1}, q_{21,2}) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_2 (q_{11,1}, q_{21,k_2}) & \phi_2 (q_{11,2}, q_{21,k_2}) & \cdots & \phi_2 (q_{11,k_1}, q_{21,k_2}) \end{pmatrix}. \end{aligned} \quad (2)$$

2. The strategy for each individual s_i is defined as a vector of weights that sum to one

$$s_1 = \begin{pmatrix} s_{11} \\ s_{12} \\ \vdots \\ s_{1k_1} \end{pmatrix} \quad s_2 = \begin{pmatrix} s_{21} \\ s_{22} \\ \vdots \\ s_{2k_2} \end{pmatrix} \quad (3)$$

3. Following the von Neumann-Morgenstern axioms - the expected value of each strategy can then be computed as a linear function of the individual choices. For example, the value of strategy 1 for individual 1 is then

$$u(s_{11}) = \sum_{j=1}^{k_2} s_{2j} \phi_1(q_{11,1}, q_{21,j}) \quad (4)$$

4. Thus, the optimal strategy for individual 1 given a strategy vector for individual 2 can be computed as

$$u(s_1^*) = \max_{s_{1i}} \sum_{i=1}^{k_1} s_{1i} \sum_{j=1}^{k_2} \phi_1(q_{11,1}, q_{21,j}) \quad (5)$$

s.t. $\sum_{i=1}^{k_1} s_{1i} = 1$

5. Following my discussion from Lecture 33 of my Computational Economics course - $\Delta_i = \left\{ s_i \in S_i : \sum_j s_{ij} = 1, s_i \geq 0 \right\}$ defines the set of possible strategies for individual i . The set of all possible strategies can be written as

$$\Delta = \prod_{i \in N} \Delta_i \subseteq R_M \quad (6)$$

6. **Definition:** We say that $s^* \in S$ is a *Nash Equilibrium* if $s^* \in \Delta$ and for all $i \in N$ and all $s_i \in \Delta_i$, $u_i(s_i, s_{-i}^*) \leq u_i(s^*)$.

7. "Note the role of the separability assumption on B_i ; it ensures that $\hat{q}_1$ and $\hat{q}_2$ are independent of a_1 and a_2 ."

- C. Suppose a relatively simple concept where individual 1 has three possible responses $q_1 = (0, 5, 10)$ and individual two has two responses $q_2 = (0, 10)$. In each case, suppose that I had a value function $\phi_i(q_1, q_2) \gg 0$ for all q_1, q_2 .

1. I could start by considering two pure strategies $s_{11} = 1$ and $s_{21} = 1$.
2. I want to know if there is a better solution for player 1. To find this out I compute $z_j = \phi_1(q_{1j}, q_{2j})$ for $j = 1, 2, 3$. I then use this to compute $g_{1j}(q_{1j}, q_{2j}) = \phi_j(q_{1j}, q_{2j}) - \phi_1(q_{11}, q_{2j})$.
3. I could then compute a new set of strategies

$$\tilde{s}_{1i} = \frac{1 + g_{1j}(q_{1j}, q_{2j})}{1 + \sum_j g(q_{1j}, q_{2j})} \quad (7)$$

II. Integration of the Firm

- A. "Of course, give a_1 and a_2 , the noncooperative equilibrium $(\hat{q}_1, \hat{q}_2)$ is unlikely to be *ex post* efficient in the sense of maximizing

$$B_1[a_1, \phi_1(q_1, q_2)] + B_2[a_2, \phi_2(q_1, q_2)] \quad (8)$$

Therefore, the two parties can gain from writing a new contract at date 1 that specifies that $q_1 = q_1(a_1, a_2)$, $q_2 = q_2(a_1, a_2)$, where these are the maximizers of 8 (if there are several maximizers, chose any pair)."

1. Making the selection of the contract nonseparable from the renegotiation $\hat{q} \equiv (\hat{q}_1, \hat{q}_2)$ and $q(a) \equiv [q_1(a), q_2(a)]$, where $a = (a_1, a_2)$.
2. The concept is to develop a *transfer price* p that reallocates the gains from renegotiation – As a starting point Grossman and Hart assume that the parties split the gain in total surplus 50/50.
3. To derive this transfer price – consider the effect on the payoff

$$\begin{aligned}
 B_1 \{ a_1, \phi_1 [q(a)] \} - p &= B_1 [a_1, \phi_1 (\hat{q})] + \frac{1}{2} (B_1 \{ a_1, \phi_1 [q(a)] \} + B_2 \{ a_2, \phi_2 [q(a)] \} \\
 &\quad - B_1 [a_1, \phi_1 (\hat{q})] - B_2 [a_2, \phi_2 (\hat{q})]) \equiv \xi(a, \hat{q}) \\
 p + B_2 \{ a_2, \phi_2 [q(a)] \} &= B_2 [a_2, \phi_2 (\hat{q})] + \frac{1}{2} (B_1 \{ a_1, \phi_1 [q(a)] \} + B_2 \{ a_2, \phi_2 [q_2(a)] \} \\
 &\quad - B_1 [a_1, \phi_1 (\hat{q})] - B_2 [a_2, \phi_2 (\hat{q})]) \equiv \xi_2(a, \hat{q})
 \end{aligned} \tag{9}$$

4. Let's break this down a little bit.

a. The "pure pricing component" is

$$\begin{aligned}
 p &= \frac{1}{2} (B_1 \{ a_1, \phi_1 [q(a)] \} + B_2 \{ a_2, \phi_2 [q_2(a)] \} \\
 &\quad - B_1 [a_1, \phi_1 (\hat{q})] - B_2 [a_2, \phi_2 (\hat{q})])
 \end{aligned} \tag{10}$$

b. The first term

$$B_1 \{ a_1, \phi_1 [q(a)] \} + B_2 \{ a_2, \phi_2 [q_2(a)] \} \tag{11}$$

is the value of the "new contract" or renegotiation agreement that takes into account each firm's initial investment.

c. Intuitively, firm 2 (the upstream firm – the one that commits to producing the input) return is now a function of how much relationship specific investment was made.

d. The second part of the pricing function is then the return under the original **nonintegrated** benefit function

$$B_1 [a_1, \phi_1 (\hat{q})] + B_2 [a_2, \phi_2 (\hat{q})] \tag{12}$$

Intuitively, this amount has to be subtracted from the benefits of the new status quo – the benefits from sharing the incremental gain from renegotiation.

e. The overall value to each firm is then the value of its actions under the new contract ($B_1 \{ a_1, \phi_1 [q(a)] \}$, $B_2 \{ a_2, \phi_2 [q(a)] \}$) plus the effect of the transfer of the gain.

- f. In this formulation the downstream firm 1 (the firm that buys the input from the upstream firm and sells to a final consumer) pays the price to the upstream firm 2 (the firm that produces the relationship specific input)

$$\begin{aligned} &\Rightarrow B_1 \{ a_1, \phi_1 [q(a)] \} - p \\ &\Rightarrow B_2 \{ a_2, \phi_2 [q(a)] \} + p \end{aligned} \quad (13)$$

5. "This is in fact the Nash bargaining solution. Note that most bargaining solutions will yield an *ex post* Pareto-optimal outcome given our assumptions that the parties have the same information and the bargaining (i.e., contract renegotiations) is costless."
6. Next we assume a_1 and a_2 chosen noncooperatively by agents at time 0 with renegotiation at time 1 – the Nash equilibrium at time 0 is a pair $(\tilde{a}_1, \tilde{a}_2) \in A_1 \times A_2$ such that

$$\begin{aligned} \xi_1(\tilde{a}_1, \tilde{a}_2, \hat{q}) &\geq \xi(a_1, \tilde{a}_2, \hat{q}) \text{ for all } a_1 \in A_1 \\ \xi_2(\tilde{a}_2, \tilde{a}_1, \hat{q}) &\geq \xi(\tilde{a}_1, a_2, \hat{q}) \text{ for all } a_2 \in A_2 \end{aligned} \quad (14)$$

Look familiar?

7. The *ex ante* surplus is then

$$B_1 \{ \tilde{a}_1, \phi_1 [q(\tilde{a})] \} + B_2 \{ \tilde{a}_2, \phi_2 [q(\tilde{a})] \} \quad (15)$$

It is important to keep track of the nuts and bolts of this solution – which a_i .

- a. For example, it is possible to show that this surplus is less than the first-best surplus. Specifically, each firm's first-order condition becomes

$$\frac{\partial \xi_i}{\partial a_i} = \frac{1}{2} \frac{\partial B_i}{\partial a_i} [a_i, \phi_i(\hat{q})] + \frac{1}{2} \frac{\partial B_i}{\partial a_i} \{ a_i, \phi_i [q(a)] \} = 0, i = 1, 2 \quad (16)$$

Note that we are assuming that the other decisions are made consistent with their first-order conditions (i.e., applying the envelope theory).

b. The solution in Equation 16 has to be compared with

$$\frac{\partial B_i}{\partial a_i} \{a_i, \phi_i[q(a)]\} = 0, i = 1, 2. \quad (17)$$

The difference being that each agent only receives half of the value of the renegotiation.

B. Case 2 – Firm 1 Control

1. In this case the downstream firm (the one buying the input and selling the final output) owns the upstream firm
2. "In this case firm 1 owns firm 2, and so manager 1 has the right to choose q_1 and q_2 at date 1. At date 1, manager 1 will now choose (q_1, q_2) to maximize ϕ_1 if no further negotiations takes place."
3. **Assumption 2.** There is a unique pair $(\bar{q}_1, \bar{q}_2)$ such that $(\bar{q}_1, \bar{q}_2)$ solves

$$\begin{aligned} &\text{Max } \phi_1(q_1, q_2) \\ &\text{s.t. } (q_1, q_2) \in Q_1 \times Q_2 \end{aligned} \quad (18)$$

- a. "The pair $(\bar{q}_1, \bar{q}_2)$ will generally not be *ex post* Pareto optimal, and so recontracting at date 1 will lead to the pair $q_1(a), q_2(a)$ as in the case of nonintegration."
- b. "We will continue to assume that the parties split the gains from renegotiation 50:50".
- c. "That is, owning an additional firm increases a manager's power only by raising his status quo utility, that is, his utility in the event of no renegotiation."
- d. "Given the 50 percent sharing rule, manager i 's final payoff is as (4)-(5) with $(\bar{q}_1, \bar{q}_2)$ replacing $(\hat{q}_1, \hat{q}_2)$."
- e. "Firm 1 control will generally lead to inefficient *ex ante* investments since

$$(\bar{q}_1, \bar{q}_2) \neq [q_1(a), q_2(a)]. \quad (19)$$

C. Case 3 – Firm 2 Control is exactly the same as Case 2.

D. Optimal Ownership Structure

1. "We saw in (9) that the inefficiency in a 's is due to the fact that manager i puts 50 percent weight on the noncooperative solution $(\bar{q}_1, \bar{q}_2)$ – which equals $(\hat{q}_1, \hat{q}_2)$ under nonintegration, $(\bar{q}_1, \bar{q}_2)$ under firm 1, and (q_1, q_2) under firm 2 control – instead of 100 percent on the cooperative solution $[q_1(a), q_2(a)]$."
 - "It is clear, therefore, that if one of the pairs $(\hat{q}_1, \hat{q}_2)$, $(\bar{q}_1, \bar{q}_2)$, (q_1, q_2) happens to be very close to $[q_1(a), q_2(a)]$, there will be little inefficiency in the a 's and the corresponding ownership structure will achieve approximately the first-best.

2. Proposition 1

- (A) Suppose that ϕ_i depends primarily on q_i in the sense that

$$\begin{aligned}\phi_1(q_1, q_2) &= \alpha_1(q_1) + \epsilon_1\beta_1(q_2) \\ \phi_2(q_1, q_2) &= \alpha_2(q_2) + \epsilon_2\beta_2(q_1)\end{aligned}\tag{20}$$

Then nonintegration yields approximately the first-best, while firm 1 and firm 2 control generally do not.

- (B) Suppose that ϕ_2 hardly depends on q_1 and q_2 in the sense that

$$\phi_2(q_1, q_2) = \alpha_2 + \epsilon_2\delta_2(q_1, q_2) \text{ where } \epsilon_2 > 0 \text{ is small.} \tag{21}$$

Then Firm 1 control yields approximately the first-best, while nonintegration and firm 2 control generally do not.

- (C) Suppose that ϕ_1 hardly depends on q_1 and q_2 in the sense that

$$\phi_1(q_1, q_2) = \alpha_1 + \epsilon_1\delta_1(q_1, q_2) \text{ where } \epsilon_1 > 0 \text{ is small.} \tag{22}$$

Then firm 2 control yields approximately the first-best, while nonintegration and firm 1 control generally do not.

3. First, what we are talking about is a small variation from the frontier in a nearly optimal way as depicted in Figure 1
4. "Proposition 1 says that if the noncontractables q_l ($l = 1$ or 2) have a small effect on firm j 's benefit B_j , it is efficient for firm i to control them."

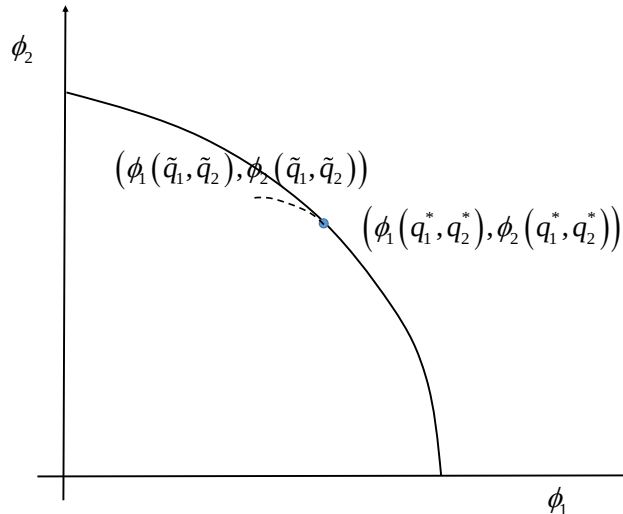


Figure 1: Small Variation from the Optimal

5. Clearly Proposition 1 side-steps the interesting issue – the case where the response is important to both firms.

6. Further Assumptions

- **Assumption 4.** Investment decisions are scalars and A_1, A_2 are intervals of the real line.
- **Assumption 5.**

$$\frac{\partial^2 B_i}{\partial \phi_i \partial a_i} [a_i, \phi_i(q_1, q_2)] > 0. \quad (23)$$

That is, marginal benefit is high when average benefit is high.

- **Assumption 6.**

$$\frac{\partial^2 B_i}{\partial a_i^2} [a_i, \phi_i(q_1, q_2)] < 0 \quad (24)$$

- **Assumption 7.** The maximizers $q_1(a)$ and $q_2(a)$ of (3) are independent of (a_1, a_2) in the relevant range; we write them as q_1^* and q_2^* .

7. From our previous derivations, we replace $\phi_i(\hat{q})$ with $\tilde{q}_i(\tilde{q}_1, \tilde{q}_2)$ (the pre-renegotiated outcome) yielding

$$\frac{1}{2} \frac{\partial B_i}{\partial a_i}(\tilde{a}_i, \tilde{\phi}_i) + \frac{1}{2} \frac{\partial B_i}{\partial a_i}(\tilde{a}_i, \phi_i^*) = 0 \quad (25)$$

The first-best investment decisions are characterized by

$$\frac{\partial B_i}{\partial a_i}(a_i^*, \phi_i^*) = 0. \quad (26)$$

The results then hinge around Assumption 5.

- a. If $a_i = a_i^*$ then

$$\begin{aligned} \frac{\partial B_i}{\partial a_i}(a_i^*, \phi_i^*) = 0 &\Rightarrow \frac{1}{2} \frac{\partial B_i}{\partial a_i}(\tilde{a}_i, \tilde{\phi}_i) + \frac{1}{2} \frac{\partial B_i}{\partial a_i}(\tilde{a}_i, \phi_i^*) > 0 \\ &\tilde{\phi} > \phi_i^*. \end{aligned} \quad (27)$$

- b. By Assumption 6

$$\tilde{a}_i \leq a_i^* \text{ as } \tilde{\phi}_i \leq \phi_i^* \quad (28)$$

8. Figure 2 depicts the various outcomes. $(\tilde{\phi}_1, \tilde{\phi}_2)$ lie on the frontier, but the optimal is (ϕ_1^*, ϕ_2^*) .
- a. Since $\bar{\phi}_1 > \phi_1^*$ and $\bar{\phi}_2 < \phi_2^* - \bar{a}_1 > a_1^*$ and $\bar{a}_2 < a_2^*$. That is under Firm 1 control firm 1 overinvests and firm 2 underinvests.
- b. At $\underline{\phi}_1 < \phi_1^*$ and the opposite result occurs.
9. "The trade-offs should now be fairly clear, Firm 1 control will be desirable when firm 1's *ex ante* investment is much more important than firm 2's (so that firm 2's underinvestment under firm 1 control is relatively unimportant) and when overinvestment by firm 1 under firm 1 control is a less severe problem than underinvestment by firm 1 as in, for example, the nonintegrated solution." – Basically, whose ox gets gored the hardest.

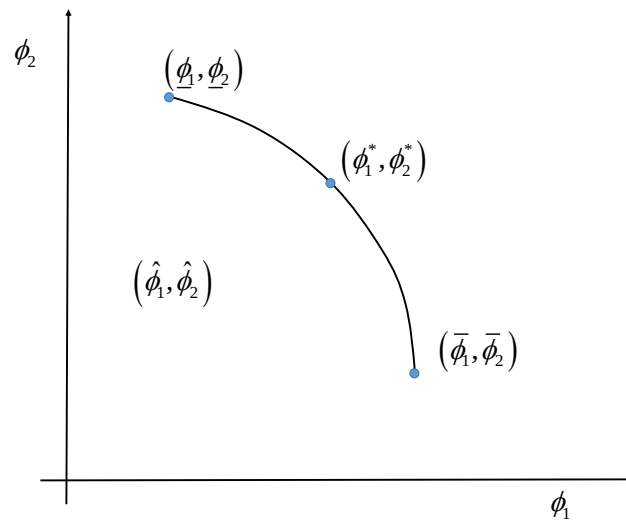


Figure 2: Significance Response for Both Firms