

Lecture II: Neoclassical Market Model for Capital II

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I. Consumption and Borrowing Decision

- A. As we developed in the last lecture, first we solve the "profit maximization problem"

$$\begin{aligned} &\text{Max } C_0 + \phi C_1 \\ &\text{s.t. } C_0^2 + \frac{C_1^2}{2} \leq 10,000. \end{aligned} \tag{1}$$

1. This results gives us a new budget constraint

$$C_0^* + \phi C_1^* = Y^*. \tag{2}$$

2. The consumer's problem then can be written as

$$\begin{aligned} &\text{Max } C_0^{0.55} C_1^{0.45} \\ &\text{s.t. } C_0 + \phi C_1 \leq Y^* \end{aligned} \tag{3}$$

3. Denoting the optimal level of consumption in period 0 in Equation 3 as $\tilde{C}_0$, the demand for capital in period 0 (optimal level of borrowing) becomes $\tilde{C}_0 - C_0^*$. This demand function is presented in Figure 1.

- B. Consider for the moment my mathematical form (an ellipse) presented in Figure 2. The part that we are interested in is very close to vertical part – it is nearly linear as depicted in Figure 3

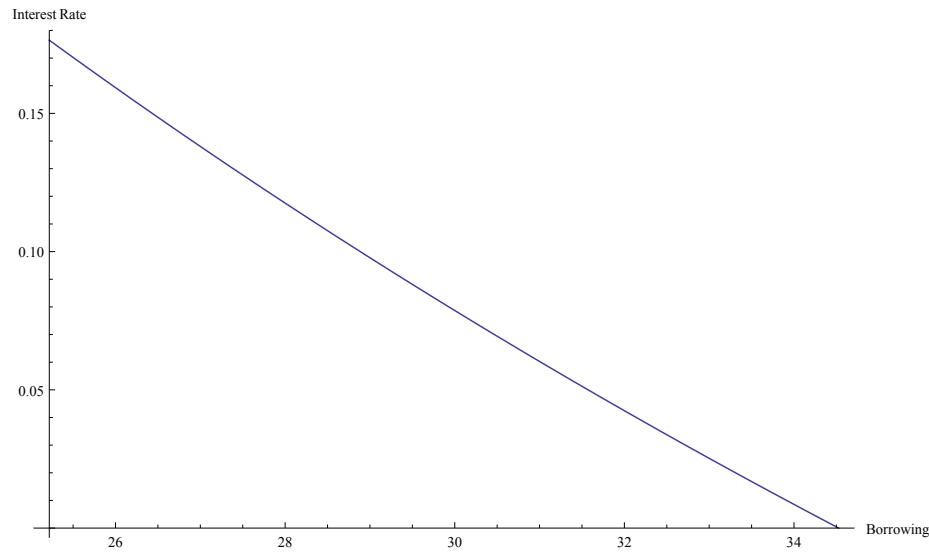


Figure 1: Demand for Capital – Borrowing

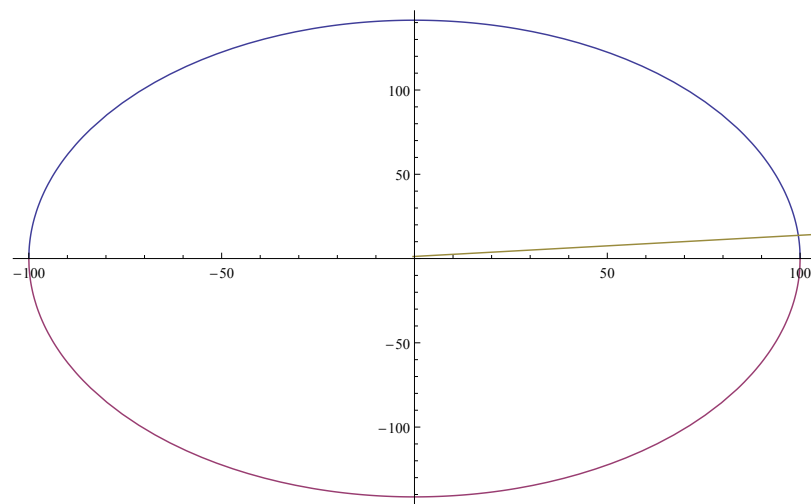


Figure 2: Production Possibility Frontier – Ellipse

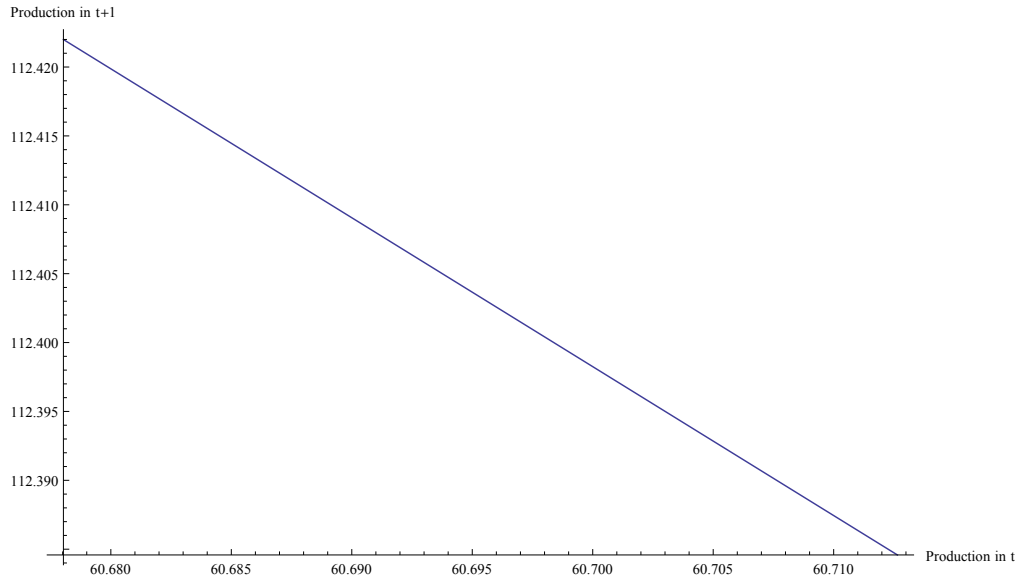


Figure 3: Relevant part of the Production Possibility Frontier

II. Closing the Market

- A. To close the market assume there is another consumer in the market that has an endowment in period 0, but not one in period 1. This consumer's problem can be simply stated as

$$\text{Max } C_0^{0.55} C_1^{0.45} \quad (4)$$

$$\text{s.t. } C_0 + \phi C_1 \leq 140.$$

1. This individual supplies capital to the market

$$\left. \begin{array}{l} 0.55 \frac{U}{C_0} - \lambda = 0 \\ 0.45 \frac{U}{C_1} - \phi \lambda = 0 \end{array} \right\} \Rightarrow \frac{0.55 C_1}{0.45 C_0} = \frac{1}{\phi} \Rightarrow C_1 = \frac{0.45}{0.55} \frac{1}{\phi} C_0 \quad (5)$$

$$C_0 + \phi \left[\frac{0.45}{0.55} \frac{1}{\phi} C_0 \right] = 140 \Rightarrow C_0 = \frac{0.55}{0.95} 140$$

$$C_1 = \frac{0.45}{0.55} \frac{0.55}{0.95} 140 \phi = \frac{0.45}{0.95} \phi$$

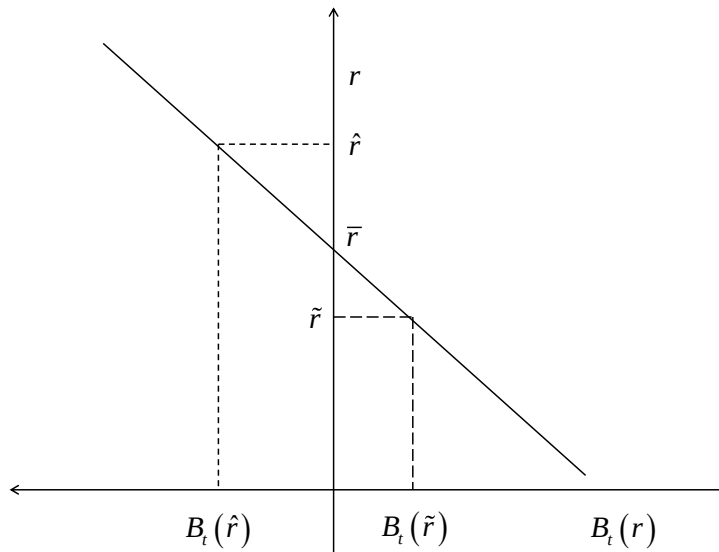


Figure 4: Demand for Capital – Borrowing

2. The interesting thing about this solution is that the current consumption level is constant. This is confirmed by the numeric solutions.
- B. Letting the demand for borrowing for the first individual become $B_1(r) = \tilde{C}_0^* - C_0^*$ and letting the demand for borrowing for the second individual become $B_2(r) = 140 - 0.55/0.95(140) = 81.05$.
1. The excess demand curve for the market is then defined as $B_1(r) + B_2(r)$.
 2. An equilibrium exists if the excess demand curve can be driven to zero

$$B_1(r) + B_2(r) = 0 \Rightarrow B_1(r) = -B_2(r) \quad (6)$$

3. In this case, the blue line in Figure 6 depicts the original equilibrium.
4. The red line in Figure 6 addresses a question raised in class – what happens if there is a change in technology? To address

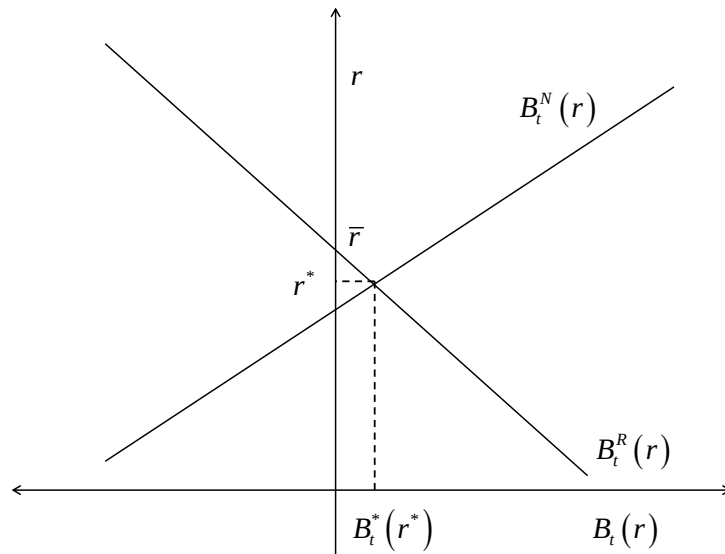


Figure 5: Market Equilibrium

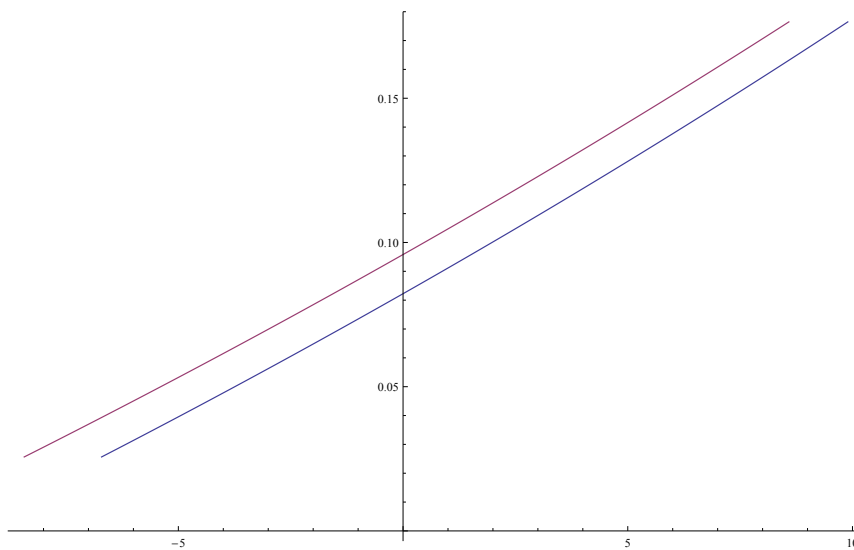


Figure 6: Market Equilibrium for Cobb Douglas examples

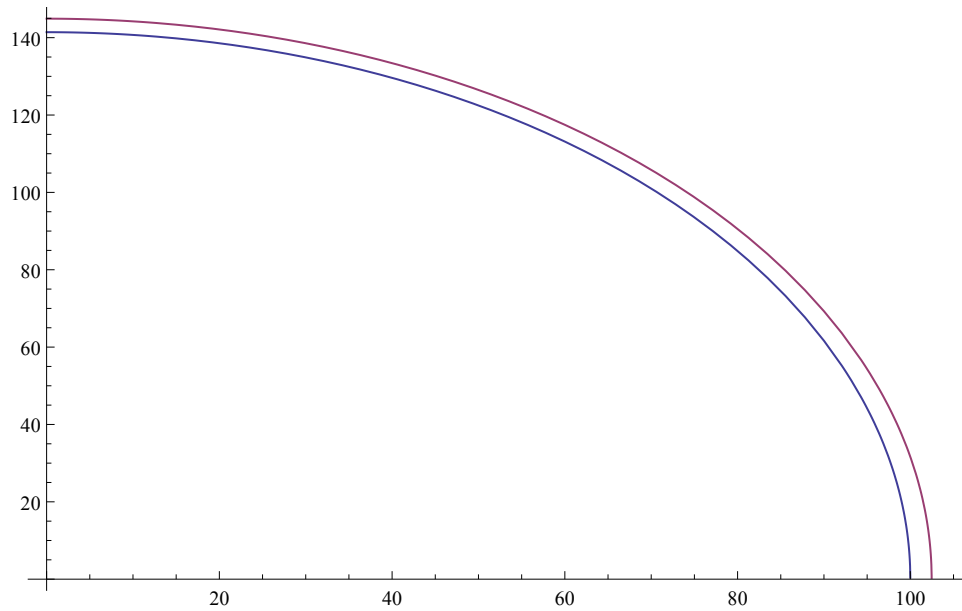


Figure 7: Outward Shift in PPC

this question, I modified the original production possibility frontier

$$C_0^2 + \frac{C_1^2}{2} \leq 10,000 + \delta. \quad (7)$$

Thus, setting $\delta = 500$ shifts the frontier outward as depicted in Figure 7.