

Nonstationary Time Series Analysis: Lecture XXXIX

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Readings

- * Greene, W.H. 2012. *Econometric Analysis* Seventh Edition (Chapter 21 pp. 942 - 971)]
- * Davidson, R. and J.G. MacKinnon. 2004. *Econometric Theory and Methods*. Oxford University Press. (Chapter 14 pp. 605 - 644)
- * Johansen, S. 1995. *Likelihood-Based Inference in Cointegrated Vector Autoregressive Model*. Oxford University Press.
- Mishra, A.K., C.B. Moss, and K.W. Erickson. 2008. The Role of Credit Constraints and Government Subsidies in Farmland Valuation in the U.S.: An Options Pricing Model Approach. *Empirical Economics* 34(2), 285-297.

Readings

- * Moss, C.B. 2006. Valuing State Level Funding for Research: Results for Florida. *Journal of Agricultural and Applied Economics* 38(1), 169-183.
- * Moss, C.B. 1992. The Cost Price Squeeze in Agriculture: An Application of Cointegration. *Review of Agricultural Economics* 14(1), 209-217.
- * Moss, C.B. and A. Schmitz. 2002. Price Behavior in the U.S. Sweetener Market: A Cointegration Approach. *Applied Economics* 34(10), 1273-1281.

Nonstationarity and Unit Roots

- In our development of autoregressive models, we considered the formulation

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \epsilon_t. \quad (1)$$

From this formulation we derived the expression

$$y_t = \alpha_0 \sum_{l=0}^{N-1} \alpha_1^l + \sum_{l=1} \alpha_1^l \epsilon_{t-l} + \epsilon_t \quad (2)$$

- In the formulation in Equation 2, if $\alpha_1 < 1$ the contribution of an error (event or outcome) dies out over time

$$\frac{\partial y_t}{\partial \epsilon_{t-k}} = \alpha_1^k \rightarrow 0 \text{ as } k \rightarrow \infty \quad (3)$$

This structure is referred to as stationary.

Random Walks

- If we assume that $\alpha_1 \rightarrow 1$, the autoregressive representation becomes a random walk

$$y_t = \alpha_0 + y_{t-1} + \epsilon_t \quad (4)$$

Any variations become permanent, from Equation 2

$$y_t = \alpha_0 \sum_{l=0}^{N-1} 1 + \sum_{l=1}^N \epsilon_{t-l} + \epsilon_t. \quad (5)$$

From Equation 3

$$\frac{\partial y_t}{\partial \epsilon_{t-k}} = 1 \quad \forall t \quad (6)$$

Differences

- Analysis of the system assumes that something is happening – For example, we want to explain something from the error term

$$y_t = y_{t-1} + \alpha x_t + \epsilon_t \quad (7)$$

- For years, the mode of analysis was to difference the system

$$y_t - y_{t-1} = \alpha x_t + \epsilon_t \quad (8)$$

This ends up being a bad idea (or at least needs some modification).

Differences – Continued

- It does allow us to discuss the estimation of significance of nonstationarity (also referred to as unit roots $\alpha_0 = 1$).
- Specifically, most of the tests for nonstationarity involve

$$z_t = y_t - y_{t-1} = \alpha_0 + \epsilon_t \quad (9)$$

Dickey-Fuller

- Consider the regression

$$y_t = \gamma y_{t-1} + \epsilon_t, \quad \epsilon_t \sim N[0, \sigma^2], \quad \text{and } \text{Cov}[\epsilon_t, \epsilon_s] = 0, \forall t \neq s \quad (10)$$

- The concept of a unit root appears related to $P[\gamma = 1]$.
 - The problem is that as $\gamma \rightarrow 1$, the variance of the equation approaches infinity – which messes with the standard t-statistics.
 - It ends up we are still interested in the t-statistic, but this t-statistic has a different **limiting distribution** which is computed empirically.
- The Dickey-Fuller test for non-stationarity is then

$$DF_\tau = \frac{\hat{\gamma} - 1}{\text{Est.Std.Error}(\hat{\gamma})} \quad (11)$$

Dickey-Fuller – Continued

- Extending the Dickey-Fuller test to the Augmented Dickey-Fuller test to deal with more complex forms of autocorrelation yields

$$y_t = \mu + \beta t + \gamma y_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots \gamma_p \Delta y_{t-p} + \epsilon_t \quad (12)$$

Cointegration

- Once we have determined that nonstationarity exists, the next question is what to do about it? How do we use this knowledge to construct models of economic relationships.
- Starting with the economic model

$$y_t = \beta x_t + \epsilon_t \quad (13)$$

Spurious Regression

- The first step is Granger, C.W.J. and P. Newbold. 1974. Spurious Regressions in Econometrics. *Journal of Econometrics* 2, 111-120.
 - Consider two variables y_t and x_t between which we hypothesize the economic relationship in Equation 13.
 - If both variables are integrated of order 1 (i.e., both variables have unit roots) – a regression of y_t on x_t may yield spurious results – have a statistically significant regression coefficient when one does not exist.

Spurious Regression – Continued

- Don't believe me – try it yourself:
 - Generate two random walk series

$$\begin{aligned}y_t &= y_{t-1} + \epsilon_t, \epsilon_t \sim N[0, 1] \\x_t &= x_{t-1} + \nu_t, \nu_t \sim N[0, 1]\end{aligned}\tag{14}$$

- Then regress

$$y_t = x_t\beta + \zeta_t\tag{15}$$

- The odds are very good that the statistical relationship will be significant.
- Further the result does not vanish as the sample size increases

Simple Linear – Two Stage Models

- Engle, R.F. and Granger, C.W.J. 1987. Co-Integration and Error Correction: Representation, Estimation, and Testing. *Econometrica* 55(2), 251-276. – The one that won the Nobel Prize for Economics.
- First, let us save ourselves from spurious regression – compute the estimated residuals from Equation 15

$$\hat{\zeta}_t = y_t - x_t \hat{\beta} \quad (16)$$

Next, compute the Dickey-Fuller statistic for ζ_t

$$\hat{\zeta}_t = \hat{\gamma} \hat{\zeta}_{t-1} + u_t \quad (17)$$

In all likelihood, these residuals will be nonstationary – in the parlance of cointegration literature a stationary cointegrating vector will not exist.

Simple Linear – Two Stage Models - Continued

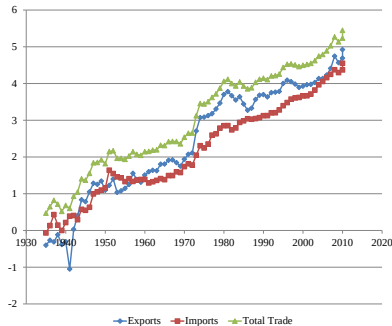
- This result points to an early "fix" – If a linear estimator can be estimated such that the implied residual is stationary, you have the long-run equilibrium relationship

$$\zeta_t = y_t - \beta x_t \ni: DF_{\tau} \text{ is small.} \quad (18)$$

- Moss, C.B. 1992. The Cost-Price Squeeze in Agriculture: An Application of Cointegration. *Review of Agricultural Economics* 14(2), 205-213. Used this approach.

Simple Linear – Two Stage Models - Continued

- The classical story is that differencing the series (as in ARIMA models) tended to remove the systematic relationship between the variables along with the unit root.



Setting Up a Cointegration

- Let's change Equation 14 to yield a cointegrating vector.
 - First, let's assume that x_t is nonstationary

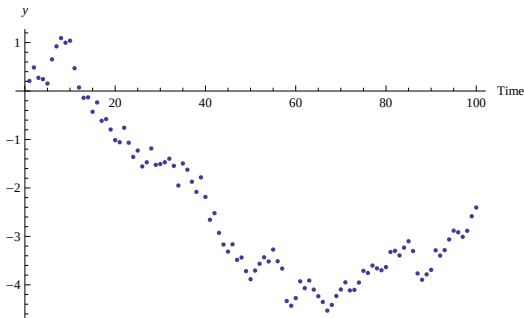
$$x_t = x_{t-1} + \epsilon_t \ni \epsilon_t \sim N[0, \sigma_\epsilon^2] \quad (19)$$

Next, let's generate y_t based on this nonstationary time series

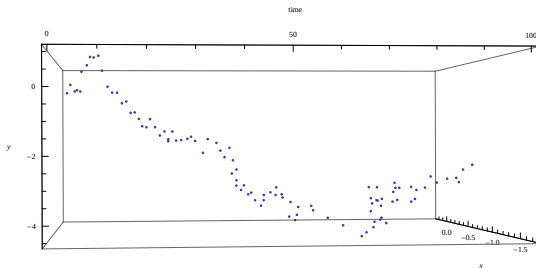
$$y_t = 0.75x_t + \nu_t \ni \nu_t \sim N[0, \sigma_\nu^2] \quad (20)$$

- Both x_t and y_t should fail the Dickey-Fuller test for stationarity.
- However, if you estimate the cointegrating vector and compute the residuals $\hat{\nu}_t$ should be stationary.

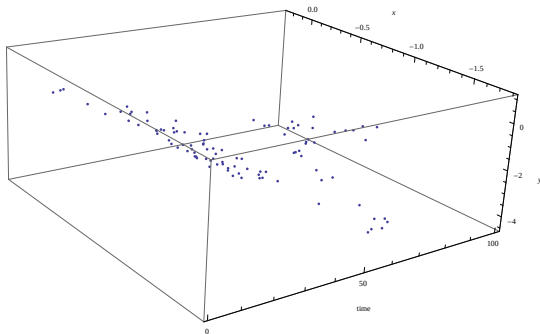
Graphics of Cointegration



Graphics of Cointegration



Graphics of Cointegration



Johansen's Approach

- My current favorite approach to cointegration is fitting a Johansen's error correction model

$$\Delta x_t = \Pi x_{t-1} \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Phi D_t + \epsilon_t \quad (21)$$

- If a cointegrating vector exists - Π is not of full-rank.
- The approach is related to Anderson's reduced rank regression.
- If Π is not of full rank at least one of its eigenvalues is zero – Thus, the test involves the statistical distribution of the eigenvalues of Π .