

Stationary Time Series Analysis: Lecture XXXVII

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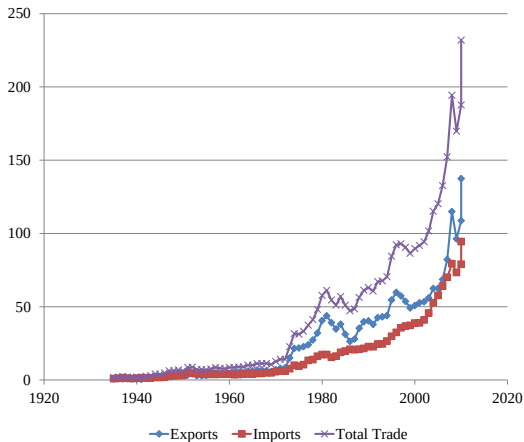
Readings – Next Time

- * Greene, W.H. 2012. *Econometric Analysis* Seventh Edition. (Chapter 20 [Section 20.1 - 20.7 pp. 903 – 924])

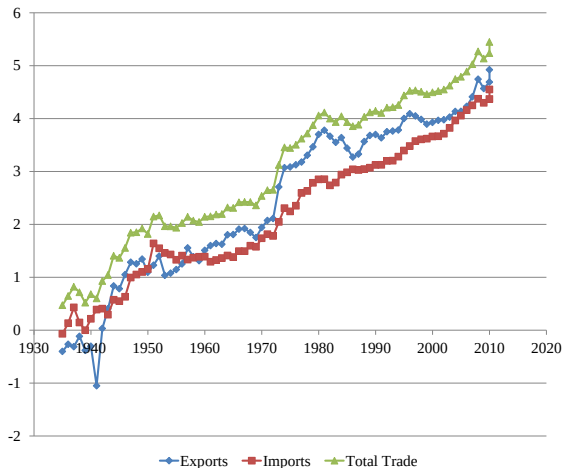
Graphic Analysis of Stationarity

- One definition of stationarity involves regular crossings of the mean.
- Consider the following datasets – Agricultural Exports, Imports, and Total Trade

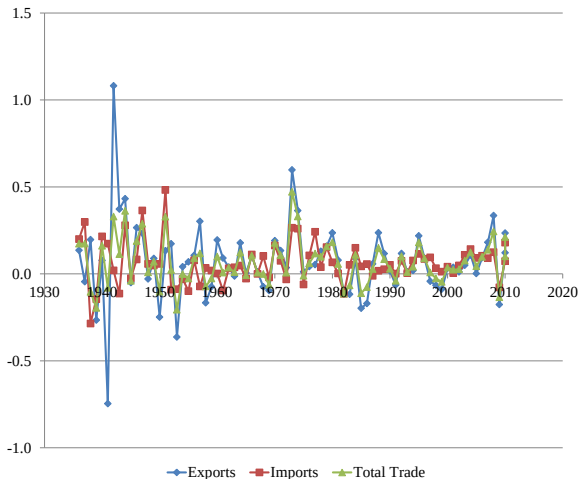
Levels Data



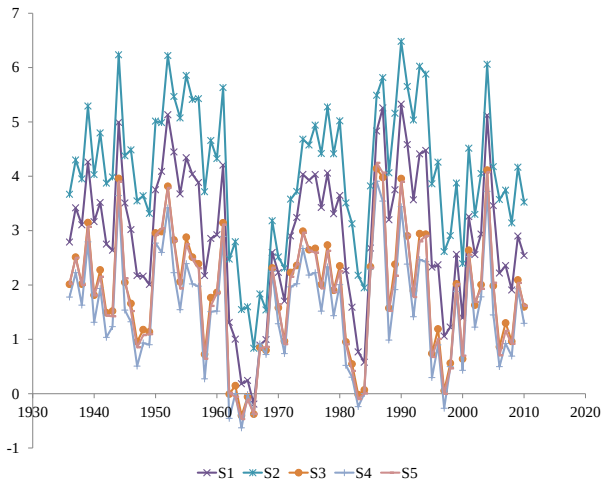
Log-Levels Data



Difference Log-Level Data



Pseudo Sample Data



Identification of Order

- The first step is to identify the order of the ARMA process.
 - In Lecture 39 we will present some more formal tests for integration, but looking at the graphs we conclude that the agricultural import and export data are log-difference stationary and the pseudo samples are stationary in levels.
 - Thus, for each series we want to identify the number of autoregressive and moving average terms.

$$ARMA(p, q) \Rightarrow y_t - \alpha_0 - \alpha_1 y_{t-1} - \alpha_2 y_{t-2} - \cdots - \alpha_p y_{t-p} = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots + \theta_q \epsilon_{t-q} \quad (1)$$

Autocorrelation and Partial Autocorrelation

- As a first step we compute the autocorrelation function – the correlation between y_t and y_{t-k} for $k = 1, \dots, \tilde{k}$.
- After the correlation matrix we compute the autocorrelation function – the partial autocorrelation between y_t and y_{t-k} such that the correlations between y_t and y_{t-l} for all $l < k$ have been removed.
 - A repeated application of our partial covariance formulation

$$\Omega_{2.1} = \Omega_2 - \Omega_{21}\Omega_{11}^{-1}\Omega_{12}$$

$$R_{2.1} = \left(\sqrt{\text{diag}[\Omega_{2.1}]} \right)^{-1} \Omega_{2.1} \left(\sqrt{\text{diag}[\Omega_{2.1}]} \right)^{-1} \quad (2)$$

Results for Pseudo Data – 1

Lag	Autocorrelation	Partial Autocorrelation
1	0.66016190	-0.03546773
2	0.41607649	-0.07881555
3	0.21797627	-0.03913904
4	0.08170292	-0.04749068
5	-0.01451626	-0.16070652

- Conclusion – AR(1)

Results for Pseudo Data – 2

Lag	Autocorrelation	Partial Autocorrelation
1	0.63209583	0.20064262
2	0.51975431	-0.10246426
3	0.33058034	-0.07603872
4	0.19630323	-0.03927551
5	0.08774398	-0.14049714

- Conclusion – Possible AR(2)

Results for Pseudo Data – 3

Lag	Autocorrelation	Partial Autocorrelation
1	0.481268221	-0.002594654
2	0.229790839	-0.055469929
3	0.068264592	-0.020399545
4	-0.003892402	-0.013624658
5	-0.030803734	-0.143029895

- Conclusion – MA(2)

Results for Pseudo Data – 3

Lag	Autocorrelation	Partial Autocorrelation
1	0.38558521	0.015878554
2	0.16222750	-0.096802124
3	-0.01351058	0.005265053
4	-0.02898519	0.001666985
5	-0.02470402	-0.127586530

- Conclusion – MA(2)

Estimation and Identification

- Estimation and lag selection in AR(p) models are relatively straightforward because we can use ordinary least squares.
- Consider applying least squares to Pseudo Data 1 using three specifications

$$\begin{aligned}y_t &= \alpha_0 + \alpha_1 y_{t-1} + \epsilon_t \\y_t &= \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \epsilon_t \\y_t &= \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \epsilon_t\end{aligned}\tag{3}$$

- In some ways, the order of the AR process should be apparent in the statistical significance of the α 's.

Pseudo Data 1 – AR Results

Parameter	AR(1)	AR(2)	AR(3)	AR(4)
α_0	0.9764 (0.2933)	1.0115 (0.3194)	1.0939 (0.3459)	1.1381 (0.3758)
α_1	0.6595 (0.0917)	0.6829 (0.1230)	0.6803 (0.1236)	0.6771 (0.1249)
α_2		-0.0354 (0.1229)	0.0184 (0.1496)	0.0195 (0.1507)
α_3			-0.0786 (0.1233)	-0.0525 (0.1495)
α_4				-0.0389 (0.1241)

Post-Estimation Statistics – Portmanteau Tests

- The next question is "Does any information remain in the residuals?"
- To analyze this question, we start by compute the autocorrelation functions for the residuals based on the AR(1) formulation

Residual Autocorrelation

Lag	Autocovariance	Autocorrelation
1	0.063	0.061
2	0.052	0.051
3	-0.006	-0.006
4	0.004	0.004
5	0.019	0.018
6	-0.122	-0.120
7	-0.190	-0.186
8	0.039	0.038
9	-0.136	-0.133

A Formal Test

- One formal test for the significance of autocovariance in the residuals is the Ljung-Box test

$$Q = n(n+2) \sum_{k=1}^h \frac{\hat{c}_k^2}{n-k} \sim \chi_h^2 \quad (4)$$

where $\hat{c}_k$ is the estimated autocorrelation and n is the number of residuals.

Introduction to Moving Average

- The estimation of moving average specifications are slightly more difficult – primarily due to changes in the variance matrix.
- Consider a ARMA(1,1)

$$y_t - \alpha_0 - \alpha_1 y_{t-1} = \epsilon_t + \theta_1 \epsilon_{t-1} \quad (5)$$

- The contemporary variance coefficient is simply σ^2 , but there is a lagged variance term $\theta^2 \sigma^2$. Hence, the overall variance becomes $(1 + \theta^2) \sigma^2$.
- There is also a recursion process in the residuals – Compute y_2 and then use the results to compute y_3 , etc.

Autoregressive Conditional Heteroscedasticity – ARCH

- As a final shot in stationary time series we consider the case where the variance changes over time.

$$\begin{aligned} y_t &= \beta x_t + \epsilon_t \\ \tilde{\sigma}_t^2 &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \alpha_3 \epsilon_{t-3}^2 \end{aligned} \quad (6)$$

- Moss, Shonkwiler, and Ford used this formulation to analyze the effect of changes in the riskiness of agriculture on the optimal debt-to-asset ratio

$$\begin{aligned} \delta_t &= \beta_0 + \beta_1 \tilde{R}_t + \beta_2 r_t + \beta_3 \hat{\sigma}_t^2 + \nu_t \\ \tilde{R}_t &= \gamma_0 + \gamma_1 R_{t-1} + \gamma_2 R_{t-2} + \epsilon_t \\ \hat{\sigma}_t^2 &= \alpha_0 + \alpha_1 |\alpha_2 \epsilon_{t-1} + \alpha_3 \epsilon_{t-2} + \alpha_{t-3}| \end{aligned} \quad (7)$$

Updated Formulation

- A recurring problem is ensuring that the estimated variance is positive. Consider the following slight reformulation

$$\epsilon_t = y_t - (\beta_0 + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \beta_3 y_{t-2} + \beta_4 x_{t-2})$$

$$\hat{\sigma}_t^2 = \alpha_0 + \begin{bmatrix} \epsilon_{t-1} & \epsilon_{t-2} & \epsilon_{t-3} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{t-1} \\ \epsilon_{t-2} \\ \epsilon_{t-2} \end{bmatrix} \quad (8)$$

- To guarantee positive variances, we use the triangular formulation

$$\hat{\sigma}_t^2 = \alpha_0 + \begin{bmatrix} \epsilon_{t-1} & \epsilon_{t-2} & \epsilon_{t-3} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{t-1} \\ \epsilon_{t-2} \\ \epsilon_{t-2} \end{bmatrix} \quad (9)$$

Likelihood Function

- The likelihood function for estimation then becomes

$$\ln(L) \propto -\frac{1}{2} \sum_{t=1}^T \left[\ln \left| \alpha_0 + \begin{bmatrix} \epsilon_{t-1} \\ \epsilon_{t-2} \\ \epsilon_{t-3} \end{bmatrix}' \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{t-1} \\ \epsilon_{t-2} \\ \epsilon_{t-3} \end{bmatrix} \right| \right. \\ \left. + \frac{(y_t - \beta_0 - \beta_1 y_{t-1} - \beta_2 x_{t-1} - \beta_3 y_{t-2} - \beta_4 x_{t-2})^2}{\alpha_0 + \begin{bmatrix} \epsilon_{t-1} \\ \epsilon_{t-2} \\ \epsilon_{t-3} \end{bmatrix}' \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{t-1} \\ \epsilon_{t-2} \\ \epsilon_{t-3} \end{bmatrix}} \right] \quad (10)$$

Readings – Next Time

- * Greene, W.H. 2012. *Econometric Analysis* Seventh Edition (Chapter 21 pp. 942 - 971])
- * Davidson, R. and J.G. MacKinnon. 2004. *Econometric Theory and Methods*. Oxford University Press. (Chapter 14 pp. 605 - 644)
- * Johansen, S. 1995. *Likelihood-Based Inference in Cointegrated Vector Autoregressive Model*. Oxford University Press.
- Mishra, A.K., C.B. Moss, and K.W. Erickson. 2008. The Role of Credit Constraints and Government Subsidies in Farmland Valuation in the U.S.: An Options Pricing Model Approach. *Empirical Economics* 34(2), 285-297.

Readings – Next Time

- * Moss, C.B. 2006. Valuing State Level Funding for Research: Results for Florida. *Journal of Agricultural and Applied Economics* 38(1), 169-183.
- * Moss, C.B. 1992. The Cost Price Squeeze in Agriculture: An Application of Cointegration. *Review of Agricultural Economics* 14(1), 209-217.
- * Moss, C.B. and A. Schmitz. 2002. Price Behavior in the U.S. Sweetener Market: A Cointegration Approach. *Applied Economics* 34(10), 1273-1281.