

Limited Dependent Variables – Part II: Lecture XXXIV

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Readings

- * Cameron, A. C. and P. K. Trivedi. 2010. *Microeconometrics Using Stata Revised Edition*. Stata Press. (Chapter 15).
- * Greene, G., C.B. Moss, and T.H. Spreen. 1997. Demand for Recreational Fishing in Tampa Bay, Florida: A Random Utility Approach. *Marine Resource Economics* 12(4), 293-305.
- * Greene, W.H. 2012. *Econometric Analysis*. Seventh Edition (Chapter 17 [Section 17.5 pp. 738 – 752])
- * Maddala, G.S. 1983. *Limited Dependent and Qualitative Variables in Econometrics* Cambridge University Press. (Chapter 3 pp. 59 - 78)

Credit Rationing Problem

- As a starting point to a fictitious dataset, assume that we have a collection of farmers who apply for a loan.
- The probability of getting a loan is assumed to be
 - An increasing function of their profitability (measured as the rate of return per acre),
 - A decreasing function of their debt-to-asset ratio,
 - An increasing function of their off-farm income, and
 - An decreasing function of their credit card debt.

Behind the Curtain

- I want to simulate this dataset (I don't have that access to ARMS and the data may not even exist there).
 - First, what are the scales of the data?
 - Return per acre say about -\$25 - \$250 (normal or semi-normal).
 - Debt-to-asset ratio say about 0.05 - 0.65 (actually probably Gamma, but an open question).
 - Off-farm income \$15,000 - \$75,000 (probably normal)
 - Credit card debt \$0 - \$25,000 (probably Gamma).
 - Using these concepts, I generate a pseudo-sample.

Simulated Sample

Credit Rationed	Profit	Debt-to-Asset	Off Farm Income	Credit Card Debt
1	115.81	0.2053	23,040.64	8,356.87
0	118.57	0.1756	14,221.16	16,662.14
0	34.96	0.1421	79,092.79	21,662.99
1	231.54	0.0972	8,220.92	11,225.39
0	133.64	0.1610	25,139.68	15,750.25
1	200.07	0.1051	60,060.65	9,628.10
1	283.69	0.2782	64,130.08	5,927.17
1	170.90	0.1016	-26,435.74	224.51
1	-104.30	0.2489	77,563.98	8,033.90
1	235.05	0.1354	43,242.79	8,902.78

Estimation

- As stated in Lecture 33 we can estimate the parameters using maximum likelihood

$$\max_{\beta} \ln (L [\beta]) = \sum_{i=1}^N [y_i \ln (f (x_i, \beta)) + (1 - y_i) \ln (1 - f (x_i, \beta))] \quad (1)$$

- Where we have several choice of $f (x_i, \beta)$.
 - One choice is the Logit function

$$f (x_i, \beta) = \frac{\exp (-x_i \beta)}{1 + \exp (-x_i \beta)} \quad (2)$$

- Another choice is the Probit function

$$f (x_i, \beta) = \int_{-\infty}^{(-x_i \beta)} \frac{1}{2\pi} \exp \left[-\frac{1}{2} z^2 \right] dz = \Phi (x_i \beta) \quad (3)$$

Estimation – Continued

- Continued
 - Older applications used simple linear models ($F(x_i, \beta) = x_i\beta$) leading to many difficulties including negative estimates of the probability.
 - More recent applications include considerations such as the asymmetry of the probability surface:

- The **Gumbel model**

$$f(x_i, \beta) = \exp[-\exp(-x_i\beta)] \quad (4)$$

- And the **Complementary Log Log Model**

$$f(x_i, \beta) = 1 - \exp[-\exp(x_i\beta)] \quad (5)$$

- Regardless of the function form, the Cramer-Rao lower bound is computed as the Hessian of Equation 1 to estimate the variance matrix for the parameters.

Estimates

Parameter	Logit Estimate	Probit Estimate
Constant	0.7683 (0.9494)	-0.2421 (0.4194)
Profitability	-0.0427 (0.0085)	-0.0111 (0.0019)
Debt-to-asset	-3.8580 (5.6239)	-1.1385 (2.4171)
Off Farm Income	-0.0002 (0.0000)	0.0000 (0.0000)
Credit Card Debt	0.0009 (0.0002)	0.0003 (0.0000)

Marginal Probabilities

- In each formulation the parameters can be used to derive the marginal probability of the exogenous variable.
- For example, in our example

$$\hat{y}_i(\beta) = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 i + \epsilon_t$$

$$F(-\hat{y}_i(\beta)) \Rightarrow \frac{\partial F(-\hat{y}_i(\beta))}{\partial x_2} \quad (6)$$

yields the marginal effect of the debt-to-asset ratio on probability of being credit constrained.

At Sample Average

- Taking the Logit model

$$\begin{aligned}
 F(a, z) &= \frac{\exp[-az]}{1 + \exp[-az]} \\
 f(a, z) &= \frac{\partial F(a, z)}{\partial z} = \frac{-a \exp[-az]}{1 + \exp[-az]} + \frac{a \exp[-az] \exp[-az]}{(1 + \exp[-az])^2} \\
 &= -\frac{a}{1 + \exp[-az]} F(a, z)
 \end{aligned}
 \tag{7}$$

	Parameter			
	Profit	Debt-to-asset	Off-Farm Income	Credit Card Debt
Marginal	0.0201	1.8146	0.0001	- 0.0004

Average Partial Effects

- The forgoing development evaluated the marginal probability at the sample mean.
- What about when one of the variables is a dummy variable (i.e., $x_{ki} = 1$ if the individual is a male and $x_{ki} = 0$ if the individual is a female).

$$APE = E_x \left[\frac{\partial E[y|x]}{\partial x} \right] \quad (8)$$

for the sample average

$$\widehat{APE} = \frac{1}{n} \sum_{i=1}^N f(x_{it} \hat{\beta}) \hat{\beta} \quad (9)$$

Average Partial Effects

Obs.	Parameter			
	Profit	Debt-to-asset	Off-Farm Income	Credit Card Debt
1	0.00768	0.69353	0.00003	-0.00017
2	0.00001	0.00097	0.00000	0.00000
3	0.00026	0.02374	0.00000	-0.00001
4	0.01041	0.93958	0.00004	-0.00023
5	0.00032	0.02920	0.00000	-0.00001
6	0.00000	0.00021	0.00000	0.00000
7	0.00000	0.00000	0.00000	0.00000
Avg.	0.00231	0.20845	0.00001	-0.00005

Goodness of Fit

- McFadden's likelihood ratio index

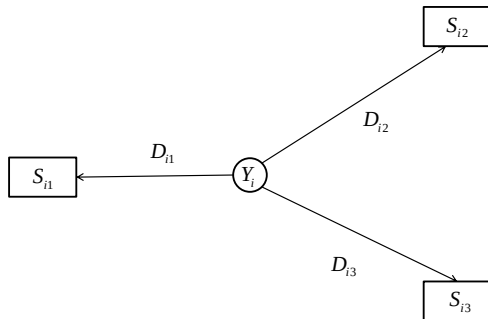
$$LRI = 1 - \frac{\ln(L)}{\ln(L_0)} \quad (10)$$

where L_0 is the binary estimate with only a constant term.

- Bewn-Akiva and Lerman (1985) and Kay and Little (1986)

$$R_{BL}^2 = \frac{1}{n} \sum_{i=1}^n \left[y_i \hat{F}_i + (1 - y_i) (1 - \hat{F}_i) \right] \quad (11)$$

Multinomial Logit



$$U_{i1} = \alpha + \beta_1 D_{i1} + \beta_2 S_{i1} + \beta_3 Y_i + \varepsilon_{i1}$$

$$U_{i2} = \alpha + \beta_1 D_{i2} + \beta_2 S_{i2} + \beta_3 Y_i + \varepsilon_{i2}$$

$$U_{i3} = \alpha + \beta_1 D_{i3} + \beta_2 S_{i3} + \beta_3 Y_i + \varepsilon_{i3}$$

Data

Obs	D_{1i}	D_{2i}	D_{3i}	S_{1i}	S_{2i}	S_{3i}	Y_i
1	5.096	8.003	3.365	2.903	6.165	5.571	416.9
2	7.781	3.614	5.183	8.774	7.501	9.031	609.6
3	1.404	6.480	5.787	4.878	3.092	5.955	1042.3
4	1.094	6.185	4.120	8.610	9.336	7.762	253.2
5	6.645	1.454	0.063	0.974	0.918	9.625	1334.1
6	0.815	6.964	2.709	4.511	5.336	5.032	1102.7

Obs	z_{1i}	z_{2i}	z_{3i}
1	0	0	1
2	0	1	0
3	1	0	0
4	1	0	0
5	0	0	1
6	1	0	0

Estimation

- Extending the Logit model

$$P[Y_i = j] = \frac{\exp(x_{ij}\theta)}{\sum_{k=1}^3 \exp(x_{ik}\theta)} \quad (12)$$

- In the forgoing sample

$$\ln(L) = \ln \left(\frac{\exp[\alpha + \beta_1 D_{31} + \beta_2 S_{31} + \beta_3 Y_1]}{\exp[\alpha + \beta_1 D_{11} + \beta_2 S_{11} + \beta_3 Y_1] + \exp[\alpha + \beta_1 D_{21} + \beta_2 S_{21} + \beta_3 Y_1] + \exp[\alpha + \beta_1 D_{31} + \beta_2 S_{31} + \beta_3 Y_1]} \right) +$$

$$\ln \left(\frac{\exp[\alpha + \beta_1 D_{22} + \beta_2 S_{22} + \beta_3 Y_2]}{\exp[\alpha + \beta_1 D_{12} + \beta_2 S_{12} + \beta_3 Y_2] + \exp[\alpha + \beta_1 D_{22} + \beta_2 S_{22} + \beta_3 Y_2] + \exp[\alpha + \beta_1 D_{32} + \beta_2 S_{32} + \beta_3 Y_2]} \right) + \dots \quad (13)$$

Readings – Next Time

- * Cameron, A. C. and P. K. Trivedi. 2010. *Microeconometrics Using Stata Revised Edition*. Stata Press. (Chapter 16).
- * Greene, W.H. 2012. *Econometric Analysis*. Seventh Edition (Chapter 19 [Section 19.1 - 19.3 pp. 833 – 860]).