

# Simulation Based Estimation and Inference – Estimating Using Simulation: Lecture XXXI

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# Readings

- \* Greene, W.H. 2012. *Econometric Analysis* Seventh Edition. (Chapter 15 pp. 603-621)
- \* Geweke, J. 1986. Exact Inference in the Inequality Constrained Normal Linear Regression Model. *Journal of Applied Econometrics* 1(2), 127-141.
- \* Moss, C.B., G.T. Livanis, and A. Schmitz. 2010. The Effect of Increased Energy Prices on Agriculture: A Differential Supply Approach. *Journal of Agricultural and Applied Economics* 42(4), 711-718.
- \* Terrell, D. 1996. Incorporating Monotonicity and Concavity in Flexible Functional Forms. *Journal of Applied Econometrics* 11(2), 179-194.

# Simulation and Inequality Constraints

- In our discussion of restrictions, we developed the estimator for

$$R\beta = r \tag{1}$$

now we want to consider the situation where

$$R\beta \geq r \tag{2}$$

- One approach would be to consider two sets of estimators:
  - Fit the unrestricted model ( $\beta = (X'X)^{-1}(X'y)$ ) and then see if  $R\beta \geq r$ .
  - If the estimated  $\beta$  failed the constraint, we could then fit the Restricted Least Squares formulation imposing the equality.
- An alternative approach would be to estimate the problem directly using simulation.

# Monotonicity

- Consider the following function

$$C_t = \beta_1 y_t + \beta_2 y_t^2 + \alpha_0 + \alpha_1 w_{1t} + \alpha_2 w_{2t} + \alpha_3 w_{3t} + \frac{1}{2} A_{11} w_{1t} w_{1t} + A_{12} w_{1t} w_{2t} + A_{13} w_{1t} w_{3t} + \frac{1}{2} A_{22} w_{2t} w_{2t} + A_{23} w_{2t} w_{3t} + \frac{1}{2} A_{33} w_{3t} w_{3t} + \epsilon_t \quad (3)$$

- We expect

$$\frac{\partial C_t}{\partial y_t} = \beta_1 + \beta_2 y_t > 0 \quad (4)$$

or that the cost of production is monotonically increasing in output.

- However,  $\beta_1 = 0.1404$  and  $\beta_2 = -0.0189$  so that at the sample mean  $\partial C_t / \partial y_t = -0.0205$ .

# Estimation by Simulation

- An approach suggested by Geweke is to estimate the parameters by simulation.
- We start from the unbiased estimates of the linear model –  
We retain the estimated  $y_t$  and  $\epsilon_t$ .

$$\begin{aligned}\hat{y}_t &= \hat{\beta} z_t \\ \hat{\epsilon}_t &= y_t - \hat{y}_t\end{aligned}\tag{5}$$

We know that these estimates are unbiased.

## Estimation by Simulation - Continued

- Next, we compute a pseudo-sample by randomly drawing  $s \in [1, T]$  and computing a new observation

$$\tilde{y}_t = \hat{y}_t + \hat{\epsilon}_s \quad (6)$$

For example, consider for the first observation we draw  $s = 5$ , the new observation would be

$$\tilde{y}_1 = \hat{y}_1 + \hat{\epsilon}_5 \quad (7)$$

- Using this new sample, we estimate  $\tilde{\beta}_k$

$$\tilde{\beta}_k = (z'z)^{-1} z'\tilde{y} \quad (8)$$

## Estimation by Simulation - Continued

| $k$ | $\beta_1$ | $\beta_2$ | $\alpha_0$ | $\dots$ | $A_{33}$ |
|-----|-----------|-----------|------------|---------|----------|
| 1   | 0.4622    | -0.0501   | 4.1505     | $\dots$ | 1.8139   |
| 2   | -0.1008   | 0.0137    | 1.6402     | $\dots$ | -7.7351  |
| 3   | -0.2607   | 0.0317    | 7.5213     | $\dots$ | -2.4595  |
| 4   | 0.0839    | -0.0083   | 0.9141     | $\dots$ | -8.0744  |
| 5   | 0.2215    | -0.0244   | 1.4438     | $\dots$ | 4.1890   |
| 6   | 0.4776    | -0.0548   | 0.5256     | $\dots$ | -14.2083 |
| 7   | 0.1079    | -0.0090   | 7.4497     | $\dots$ | -1.8015  |
| 8   | 0.1774    | -0.0208   | 4.1989     | $\dots$ | -0.7079  |
| 9   | 0.1556    | -0.0063   | -8.8773    | $\dots$ | -7.0491  |
| 10  | 0.1828    | -0.0158   | 4.8213     | $\dots$ | -1.4746  |
| 11  | 0.1671    | -0.0146   | 7.6192     | $\dots$ | 3.3963   |
| 12  | 0.2514    | -0.0212   | 13.8360    | $\dots$ | 1.4287   |
| 13  | 0.1553    | -0.0160   | 9.4106     | $\dots$ | 0.4450   |
| 14  | -0.0152   | 0.0084    | 3.7827     | $\dots$ | 3.0703   |

# Integration of the Likelihood Function

- Greene's description of this starts with a Poisson model

$$p(y_{it}|x_{it}, u_i) = \frac{\exp(-\mu_{it}) \mu_{it}^{y_{it}}}{y_{it}!} \quad (9)$$

$$\mu_{it} = \exp(x'_{it}\beta + u_i)$$

In this example,  $u_i$  is a random individual effect which confounds the estimation.

- The log-likelihood function becomes

$$\ln L = \sum_{i=1}^N \ln \left[ \prod_{t=1}^{T_i} \frac{\exp(-\mu_{it}) \mu_{it}^{y_{it}}}{y_{it}!} \right] \quad (10)$$

$$\mu_{it} = \exp(x'_{it}\beta + u_i)$$

## Integration of the Likelihood Function – Continued

- Taking the expectation (to integrate out the nuisance factor)

$$p(y_{i1}, y_{i2}, \dots, y_{i,T_i} | X_i) = \int_{u_i} p(y_{i1}, y_{i2}, \dots, y_{i,T_i} | X_i, u_i) f(u_i) \quad (11)$$

# Normality

- Assuming Normality

$$p(y_{i1}, y_{i2}, \dots, y_{iT_i} | X_i) = \int_{-\infty}^{\infty} \left[ \prod_{t=1}^{T_i} \frac{\exp(-\mu_{it}) \mu_{it}^{y_{it}}}{y_{it}!} \right] \frac{1}{\sigma} \phi\left(\frac{u_i}{\sigma}\right) du_i \quad (12)$$

- The log-likelihood function then becomes

$$\ln(L) = \sum_{i=1}^N \left\{ \int_{-\infty}^{\infty} \left[ \prod_{t=1}^{T_i} \frac{\exp(-\mu_{it}) \mu_{it}^{y_{it}}}{y_{it}!} \right] \frac{1}{\sigma} \phi\left(\frac{u_i}{\sigma}\right) du_i \right\} \quad (13)$$

$$\mu_{it} = \exp(x'_{it}\beta + u_i)$$

- \* Greene, W.H. 2012. *Econometric Analysis* Seventh Edition.  
(Chapter 16 pp. 655-680)
- \* Albert, J. 2009. *Bayesian Computation with R*.