

Maximum Likelihood Part III – Lagrange Multiplier Test Lecture XXIX

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Readings

- * Cramer, J.S. 1986. *Econometric Applications of Maximum Likelihood Methods*. Cambridge University Press
- * Greene, W.H. 2012. *Econometric Analysis* Seventh Edition. (Chapter 14 pp. 509 598)

The Research Question

- There are several ways to identify a problem:
 - I just studied this to present the material to the class.
 - I just read this in a journal (I want to prove that I read the journals).
 - I did it for the money (chasing grants).
 - I need to do something for money I have already taken.
 - This is an important problem.
- One of the persistent arguments in the political arena has been the subsidies (typically by tax credits) for ethanol production.
 - Causing higher food prices.
 - Causes higher deficits.
 - Actually shifting the relative price of meat (livestock) relative to other foods.
- **Research Question:** What is the elasticity of supply of corn (crops) and livestock.

Data

- Dr. V. Eldon Ball at the Economic Research Service of the United States Department of Agriculture has compiled data on agricultural productivity over the past 20 years.
 - Most of his measures have been Total Factor Productivity

$$d \left(\frac{Q_{output,t}}{Q_{input,t}} \right) = \frac{\left[\left(\frac{y'_t p_t}{y'_{t-1} p_t} \right) \left(\frac{y'_t p_{t-1}}{y'_{t-1} p_{t-1}} \right) \right]^{1/2}}{\left[\left(\frac{x'_t w_t}{x'_{t-1} w_t} \right) \left(\frac{x'_t w_{t-1}}{x'_{t-1} w_{t-1}} \right) \right]^{1/2}} \quad (1)$$

- Applying this formula give a number (i.e., 1.03) implying that we are getting relatively more output from our aggregate inputs.
- Another issue is the possibility of changes in the quality of inputs and outputs over time.

Data – Continued

- These indices can be used to generate aggregate outputs and inputs.
 - Consider the construction of a price index for crop outputs (where p_t^c is the set of crop outputs)

$$P_t^C = \left[\left(\frac{p_t^{c'} y_t^c}{p_{t-1}^{c'} y_t^c} \right) \left(\frac{p_t^{c'} y_{t-1}^c}{p_{t-1}^{c'} y_{t-1}^c} \right) \right]^{1/2} \quad (2)$$

- Again, this procedure yields a “chained” index of the percent change in prices. We can then choose a base year and generate a price sequence using this “chained percentage change” (say $\tilde{p}_t^C$).
- Using this price sequence, we can then derive a series of output quantities

$$\tilde{Q}_t^C = \frac{p_t^{c'} y_t^c}{\tilde{p}_t^C} \quad (3)$$

Data – Continued

- Price and Quantity of Livestock Outputs
- Price and Quantity of Crop Outputs
- Price and Quantity of Farm Related Services
- Price and Quantity of Hired Farm Labor
- Price and Quantity of Own Labor
- Price and Quantity of Capital
- Price and Quantity of Farmland
- Price and Quantity of Materials

Static Translog Cost Function

$$\begin{aligned}
 C^* &= \alpha_0 + \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}' \begin{bmatrix} \ln(w_1) - \ln(w_5) \\ \ln(w_2) - \ln(w_5) \\ \ln(w_3) - \ln(w_5) \\ \ln(w_4) - \ln(w_5) \end{bmatrix} + \\
 &\frac{1}{2} \begin{bmatrix} \ln(w_1) - \ln(w_5) \\ \ln(w_2) - \ln(w_5) \\ \ln(w_3) - \ln(w_5) \\ \ln(w_4) - \ln(w_5) \end{bmatrix}' \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{12} & A_{22} & A_{23} & A_{24} \\ A_{13} & A_{23} & A_{33} & A_{34} \\ A_{14} & A_{24} & A_{34} & A_{44} \end{bmatrix} \begin{bmatrix} \ln(w_1) - \ln(w_5) \\ \ln(w_2) - \ln(w_5) \\ \ln(w_3) - \ln(w_5) \\ \ln(w_4) - \ln(w_5) \end{bmatrix} + \\
 &\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}' \begin{bmatrix} \ln(y_1) \\ \ln(y_2) \\ \ln(y_3) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \ln(y_1) \\ \ln(y_2) \\ \ln(y_3) \end{bmatrix}' \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix} \begin{bmatrix} \ln(y_1) \\ \ln(y_2) \\ \ln(y_3) \end{bmatrix} \\
 &\begin{bmatrix} \ln(w_1) - \ln(w_5) \\ \ln(w_2) - \ln(w_5) \\ \ln(w_3) - \ln(w_5) \\ \ln(w_4) - \ln(w_5) \end{bmatrix}' \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & \Gamma_{34} \end{bmatrix} \begin{bmatrix} \ln(y_1) \\ \ln(y_2) \\ \ln(y_3) \end{bmatrix}
 \end{aligned} \tag{4}$$

Dynamic Translog

$$\ln(C_t) = m \ln(C_t^*) + (1 - m) \ln(C_{t-1}^*) + (1 - m) \times$$

$$\left(\sum_{i=1}^4 S_{i,t-1} \ln(w_{it}) - \sum_{i=1}^3 S_{i,t-1}^* \ln(w_{i,t-1}) \right) +$$

$$\sum_{i=1}^4 \sum_{j=1}^4 b_{ij} (S_{j,t-1}^* - S_{j,t-1}) \ln(w_{it}) \quad (5)$$

$$S_t = m S_t^* + (1 - m) S_{t-1} + B (S_{t-1}^* - S_{t-1})$$

Approach

- What to do next?
 - Estimate the simple model.
 - Estimate the dynamic model.
- Always start with the simplest model possible – Unless you are just trying to prove that you can do it.

Static Estimates

- Using Maximum Likelihood:

Par	Est	Std.Dev	Par	Est	Std.Dev	Par	Est	Std.Dev
α_0	209.6096	69.4983	A_{33}	0.1239	0.0051	Γ_{11}	-0.0214	0.0115
α_1	0.3160	0.0842	A_{34}	-0.0238	0.0032	Γ_{12}	-0.0157	0.0068
α_2	2.0939	0.2391	A_{44}	0.1288	0.0054	Γ_{13}	0.0144	0.0025
α_3	0.0638	0.1784	β_1	-9.9660	21.8287	Γ_{14}	-0.1235	0.0335
α_4	1.9217	0.1587	β_2	-16.7311	14.4821	Γ_{21}	-0.0717	0.0193
A_{11}	0.0312	0.0233	β_3	-12.3863	7.4679	Γ_{22}	0.0220	0.0065
A_{12}	-0.0120	0.0231	B_{11}	-6.2603	5.1134	Γ_{23}	-0.0349	0.0292
A_{13}	-0.0145	0.0021	B_{12}	5.5229	3.6551	Γ_{24}	0.0666	0.0182
A_{14}	-0.0120	0.0012	B_{13}	2.1385	1.0354	Γ_{31}	-0.0388	0.0061
A_{22}	0.0704	0.0245	B_{22}	-2.8105	2.7197	Γ_{32}	-0.1131	0.0284
A_{23}	-0.0096	0.0053	B_{23}	-1.4427	0.4400	Γ_{33}	-0.0248	0.0187
A_{24}	-0.0127	0.0038	B_{33}	0.4689	0.3764	Γ_{34}	0.0242	0.0061

Static Estimates – Continued

- Problems with the estimates:
 - The A matrix is not negative definite (inconsistent with input demand curves)

$$A = \begin{bmatrix} 0.0312 & -0.0120 & -0.0145 & -0.0120 \\ -0.0120 & 0.0704 & -0.0096 & -0.0127 \\ -0.0145 & -0.0096 & 0.1239 & -0.0238 \\ -0.0120 & -0.0127 & -0.0238 & 0.1288 \end{bmatrix} \quad (6)$$

- The B matrix is not positive definite (inconsistent with upward sloping marginal cost curves)

$$B = \begin{bmatrix} -6.2603 & 5.5229 & 2.1385 \\ 5.5229 & -2.8105 & -1.4427 \\ 2.1385 & -1.4427 & 0.4689 \end{bmatrix} \quad (7)$$

Lagrange Multiplier Test

- Before we go to the more complicated model – should we test whether the more elaborate model would be informative?
- To do this, we use the Lagrange Multiplier test which is based on the likelihood function for the more complicated model at the restricted (static estimates).

$$LM = \nabla_{\theta} L(\theta_0) [\nabla_{\theta\theta}^2 L(\theta_0)]^{-1} [\nabla_{\theta} L(\theta_0)]' = 198.66 \sim \chi_{53-36}^2$$

(8)

Dynamic Estimates

Par	Est	Std.Dev	Par	Est	Std.Dev	Par	Est	Std.Dev
α_0	0.9319	92.5696	B_{11}	-17.9162	6.7795	m	1.0000	0.0240
α_1	0.4241	0.0797	B_{12}	13.0596	4.6386	b_{11}	-0.5080	0.1097
α_2	2.0104	0.2082	B_{13}	0.5333	1.1944	b_{22}	-0.9089	0.1122
α_3	0.0864	0.1081	B_{22}	-6.7846	3.3416	b_{33}	-0.4770	0.0831
α_4	1.4962	0.1186	B_{23}	-1.6300	0.5230	b_{44}	-0.0093	0.1146
A_{11}	0.0787	0.0315	B_{33}	0.6155	0.4699	b_{12}	-0.0259	0.0387
A_{12}	-0.0604	0.0312	Γ_{11}	-0.0399	0.0079	b_{13}	0.0340	0.0507
A_{13}	-0.0039	0.0035	Γ_{12}	0.0027	0.0056	b_{14}	-0.0800	0.0267
A_{14}	-0.0133	0.0013	Γ_{13}	0.0022	0.0023	b_{21}	0.1353	0.3237
A_{22}	0.1126	0.0326	Γ_{14}	-0.1584	0.0211	b_{23}	-0.1999	0.1630
A_{23}	-0.0028	0.0066	Γ_{21}	0.0023	0.0149	b_{24}	-0.2737	0.0883
A_{24}	-0.0282	0.0034	Γ_{22}	-0.0231	0.0054	b_{31}	0.2575	0.1605
A_{33}	0.0926	0.0067	Γ_{23}	0.0012	0.0121	b_{32}	0.3369	0.0601
A_{34}	-0.0148	0.0020	Γ_{24}	-0.0013	0.0088	b_{34}	0.1225	0.0410
A_{44}	0.1224	0.0044	Γ_{31}	0.0007	0.0039	b_{41}	-1.3817	0.4151
β_1	50.7532	30.1211	Γ_{32}	-0.0888	0.0220	b_{42}	-0.3250	0.1631
β_2	-55.7826	19.3461	Γ_{33}	0.0063	0.0163	b_{43}	-0.0942	0.2111
β_3	7.1782	8.9285	Γ_{34}	-0.0001	0.0066			

Dynamic Estimate – Continued

Par	Static		Dynamic	
	Est	Std.Dev	Est	Std.Dev
A_{11}	0.0312	0.0233	0.0787	0.0315
A_{12}	-0.0120	0.0231	-0.0604	0.0312
A_{13}	-0.0145	0.0021	-0.0039	0.0035
A_{14}	-0.0120	0.0012	-0.0133	0.0013
A_{22}	0.0704	0.0245	0.1126	0.0326
A_{23}	-0.0096	0.0053	-0.0028	0.0066
A_{24}	-0.0127	0.0038	-0.0282	0.0034
A_{33}	0.1239	0.0051	0.0926	0.0067
A_{34}	-0.0238	0.0032	-0.0148	0.0020
A_{44}	0.1288	0.0054	0.1224	0.0044

Dynamic Estimate – Continued

Par	Static		Dynamic	
	Est	Std.Dev	Est	Std.Dev
B_{11}	-6.2603	5.1134	-17.9162	6.7795
B_{12}	5.5229	3.6551	13.0596	4.6386
B_{13}	2.1385	1.0354	0.5333	1.1944
B_{22}	-2.8105	2.7197	-6.7846	3.3416
B_{23}	-1.4427	0.4400	-1.6300	0.5230
B_{33}	0.4689	0.3764	0.6155	0.4699

Readings - Next Time

- * Greene, W.H. 2012. *Econometric Analysis* Seventh Edition. (Chapter 15 pp. 603 652)