

Generalized Method of Moments – Part II: Lecture XXVI

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Readings

- * Greene, W.H. 2012. *Econometric Analysis* Seventh Edition. (Chapter 13 pp. 455 507)
- * Hall, A. 2005. *Generalized Method of Moments*. Oxford University Press (Chapters 1 3 pp. 1 114)

System Instrumental Variables

- As developed in our discussion of Two-Stage-Least-Squares (2SLS) and Three-Stage-Least-Squares (3SLS), the system instrumental variables estimator is developed from the same Generalized Instrumental Variables Estimator (GIVE) that we use to construct the Generalized Method of Moments estimator.
- Let us return to the two equation supply and demand equilibrium

$$\begin{aligned} q_1^S &= \beta_{10} + \gamma_{11}p_1 + \beta_{11}p_2 + \beta_{12}w_1 + \beta_{13}w_2 + \epsilon_1 \\ q_1^D &= \beta_{20} + \gamma_{21}p_1 + \beta_{24}y + \beta_{25}p_3 + \epsilon_2 \end{aligned} \quad (1)$$

System of Equations

- As discussed in the last lecture, I need to set up the objective function

$$Q_T(\beta) = \left\{ \frac{1}{T} u(\beta)' Z \right\} W_T \left\{ \frac{1}{T} Z' u(\beta) \right\} \quad (2)$$

- From Equation 1 we define the $u(\beta)$ as a 2×1 vector (i.e., a stacked system of equations)

$$\begin{aligned} u_{1t}(\beta) &= q_{1t} - (\beta_{10} + \gamma_{11}p_{1t} + \beta_{11}p_{2t} + \beta_{12}w_{1t} + \beta_{13}w_{2t}) = \epsilon_{1t} \\ u_{2t}(\beta) &= q_{2t} - (\beta_{20} + \gamma_{21}p_{1t} + \beta_{24}y_t + \beta_{25}p_{3t}) = \epsilon_{2t} \end{aligned} \quad (3)$$

System of Equations - continued

- As in our discussion of 3SLS, we reconstruct the Y and X matrices

$$y = \begin{bmatrix} y_{11} \\ y_{21} \\ \vdots \\ y_{1N} \\ y_{2T} \end{bmatrix}$$

(4)

$$X = \begin{bmatrix} 1 & p_{11} & p_{21} & w_{11} & w_{21} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & p_{11} & y_1 & p_{31} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & p_{1T} & p_{2T} & w_{1T} & w_{2T} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & p_{1T} & y_T & p_{3T} \end{bmatrix}$$

Instruments for the System

- Next, we retain the same instruments we used in the 2SLS formulation (p_2 , w_1 , w_2 , y , p_3 and a constant).
- The only wrinkle is that we need to consider these as potentially different instruments for each equation.
 - Thus, we start with

$$z = \begin{bmatrix} 1 & p_{2t} & w_{1t} & w_{2t} & y_t & p_{3t} \end{bmatrix}_{t=1}^T \quad (5)$$

- Then we take the Kronecker product of the instrument matrix

$$\tilde{z} = z \otimes I_{2 \times 2} \quad (6)$$

- In this case (2SLS), we know something about W_T

$$\hat{W}_T = I_{2 \times 2} \otimes (z'z/T)^{-1} \quad (7)$$

Instruments for the System - Continued

- In this case the GIVE estimator explicitly contains the generalized instrumental estimator case

$$Q_T(\beta) \Rightarrow \tilde{Z}' \hat{W}_T \tilde{Z} \Rightarrow \begin{bmatrix} Z(Z'Z/T)^{-1}Z' & 0 \\ 0 & Z(Z'Z/T)^{-1}Z' \end{bmatrix} \quad (8)$$

- Therefore

$$Q_T(\beta) = \frac{1}{T^2} [u(\beta)' Z] \hat{W}_T [Z' u(\beta)]$$

$$\Rightarrow \begin{bmatrix} (y_1 - X_1\beta_1)' Z(Z'Z/T)^{-1}Z' (y_1 - X_1\beta_1) & 0 \\ 0 & (y_2 - X_2\beta_2)' Z(Z'Z/T)^{-1}Z' (y_2 - X_2\beta_2) \end{bmatrix} \quad (9)$$

- So you have two separate 2SLS regressions

Third Stage - Covariance Between Equations

- Again, unstacking the estimated residuals yields $\epsilon \in M_{T \times 2}$, and the variance between equations can be estimated as

$$\tilde{V} = \frac{1}{T} (\epsilon' \epsilon) \quad (10)$$

- So a new estimate of the W_T matrix becomes

$$\tilde{W}_T = \left[\frac{1}{T} (\epsilon' \epsilon) \right]^{-1} \otimes (z' z / T)^{-1} \quad (11)$$

- Given the linear form of this problem the solution to Equation 2 becomes

$$\hat{\beta}_{3SLS} = \left(X' \tilde{Z} \tilde{W}_T \tilde{Z}' X \right)^{-1} \left(X' \tilde{Z} \tilde{W}_T Z' y \right) \quad (12)$$

Generalized Method of Moments

- Given that the GMM is estimated with a GIVE estimator, little change should be expected in implementing the estimator.
- In fact, let us consider the same application, only appending some "weak instruments"

$$z = \begin{bmatrix} 1 & p_{2t} & w_{1t} & w_{2t} & y_t & p_{3t} & p_{2t}^2 & p_{2t}w_{1t} & p_{2t}w_{2t} & \cdots \end{bmatrix}_{t=1}^T \quad (13)$$

- Again, use the covariance matrix for the instruments to estimate the residuals and then compute the variance of the first stage to reestimate the GMM model.

Estimation Results

Parameter	2SLS	3SLS	GMM ₁	GMM ₂
β_{10}	2.920	1.971	-1.687	-1.829
γ_{11}	0.404	0.439	0.427	0.420
β_{11}	-0.193	-0.194	-0.243	-0.248
β_{12}	-0.989	-0.255	-0.352	-0.354
β_{13}	0.353	-0.192	0.613	0.661
β_{20}	-3.817	-4.053	-7.823	-7.874
γ_{12}	0.351	0.376	-0.085	-0.088
β_{24}	0.013	0.012	-0.040	-0.041
β_{25}	0.167	0.167	2.152	2.171

Readings

- * Cramer, J.S. 1986. *Econometric Applications of Maximum Likelihood Methods*. Cambridge University Press
- * Greene, W.H. 2012. *Econometric Analysis* Seventh Edition. (Chapter 14 pp. 509 598)