

Applying Ordinary Least Squares on the Computer

- Gauss: Lecture XI

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Readings

- * Cameron, A. C. and P. K. Trivedi. 2010. *Microeconometrics Using Stata* Revised Edition. Stata Press. (Chapters 1 - 2).
- * Morgan, M.S. 1992. *The History of Econometric Ideas*. Cambridge University Press. (Chapter 1: Sunspots and Venus Theories of the Business Cycle pp. 18 - 39).

Empirical Models of Supply

- **Primal Approach:** Estimate the production functions and derive optimizing behavior.
- **Dual Approach:** Estimate the response of decision makers to changes in input and output prices - Flexible functional forms.
- **Differential Approach:** Estimate the implications of changes in profit-maximizing behavior.

Singe Product Firm

- Using the change in first-order conditions, Theil derived an empirical specification for the quantity of each input used by the firm.

$$f_{it} d \ln(x_{it}) = \theta_i d \ln(Q_t) + \sum_{j=1}^n \pi_{ij} d \ln(w_{jt}) + \epsilon_{it} \quad (1)$$

- f_{it} is the share of each input in the cost of production.
- x_{it} is the quantity of each input.
- Q_t is the total output produced (usually a quantity index).
- w_{jt} is the price of each input.

System of Input Equations

- This specification leads to a system estimation problem. Specifically, the four input case we have been discussion can be written as:

$$\begin{bmatrix} \bar{f}_{1t} Dx_{1t} \\ \bar{f}_{2t} Dx_{2t} \\ \bar{f}_{3t} Dx_{3t} \\ \bar{f}_{4t} Dx_{4t} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} DQ_t + \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} \\ \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} \\ \pi_{41} & \pi_{42} & \pi_{43} & \pi_{44} \end{bmatrix} \begin{bmatrix} Dw_{1t} \\ Dw_{2t} \\ Dw_{3t} \\ Dw_{4t} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{4t} \end{bmatrix} \quad (2)$$

- Where $\bar{f}_{it} = \frac{1}{2} (f_{i,t} + f_{i,t-1})$ and $Dz_{it} = \ln(z_{i,t}) - \ln(z_{i,t-1})$

Estimation Restrictions

- The first restriction is statistical - the left hand side variables sum to an identity.
 - Thus, the variance matrix is singular.
 - We could either estimate with a generalized inverse or typically we simply drop an equation.
- Second, given that we are estimating a cost function (of sorts) and cost functions are homogenous of degree one

$$\begin{aligned}\sum_{j=1}^n \theta_j &= 1 \\ \sum_{j=1}^n \pi_{ij} &= 0 \quad \forall i\end{aligned}\tag{3}$$

The first equality is generated by the singularity of the variance matrix. The second must be imposed.

Estimation Restrictions - Continued

- Third, by Young's theorem the π_{ij} s are symmetric ($\pi_{ij} = \pi_{ji} \forall i, j$).

Simple Linear Model with Gauss

- Consider the first equation in the system

$$\bar{f}_{1t}D(x_{1t}) = \theta_1 DQ_t + \pi_{11} Dw_{1t} + \pi_{12} Dw_{2t} + \pi_{13} Dw_{3t} + \pi_{14} Dw_{4t} + \epsilon_{1t} \quad (4)$$

- I haven't found a Gauss macro to load the data from csv.
 - Loading in ASCII - use scientific notation for accuracy.

Restricted Least Squares

- We will develop linear restrictions more completely next week, but we want to define an R matrix to impose

$$\sum_{j=1}^n \pi_{1j} = 0 \Rightarrow R\beta = r \Rightarrow R = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad r = 0 \quad (5)$$

- The restricted least squares regression is then

$$\beta_R = \beta - (X'X)^{-1} R' \left[R (X'X)^{-1} R' \right]^{-1} (R\beta - r) \quad (6)$$

Restricted Least Squares - Continued

Unrestricted	Restricted
0.002225	0.001749
0.000964	0.002745
-0.009508	-0.005099
-0.000923	-0.000376
0.001870	0.010341
-0.002324	-0.004866

System Restrictions

- Another way to impose homogeneity is to compute $Dw_{it}^* = Dw_{it} - Dw_{nt}$.
- In addition, because of the singularity, we reduce the system to three equations.

$$\begin{bmatrix} \bar{f}_{1t} Dx_{1t} \\ \bar{f}_{2t} Dx_{2t} \\ \bar{f}_{3t} Dx_{3t} \end{bmatrix} = \begin{bmatrix} \theta_1 & \pi_{11} & \pi_{12} & \pi_{13} \\ \theta_2 & \pi_{12} & \pi_{22} & \pi_{23} \\ \theta_3 & \pi_{13} & \pi_{23} & \pi_{33} \end{bmatrix} \begin{bmatrix} DQ_t \\ Dw_{1t}^* \\ Dw_{2t}^* \\ Dw_{3t}^* \end{bmatrix} + \epsilon_t \quad (7)$$

Symmetry Restrictions - Continued

- Notice that the symmetry restrictions have already been imposed.
- To estimate this system we return to the examples in the Kronecker product

$$\tilde{\beta} = [(X \otimes I_{3 \times 3})' (X \otimes I_{3 \times 3})]^{-1} [(X \otimes I_{3 \times 3})' \text{vecr}(y)] \quad (8)$$

- To simplify the notation, let $\tilde{X} = X \otimes I_{3 \times 3}$.
- Applying the restricted least squares formulation we have

$$\beta_R = \beta - (\tilde{X}'\tilde{X})^{-1} R' \left[R (\tilde{X}'\tilde{X})^{-1} R' \right]^{-1} (R\beta - r) \quad (9)$$

Symetry Restrictions - Continued

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \end{bmatrix} \quad (10)$$

$$r = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

Symetry Restrictions - Continued

$$\beta = \begin{bmatrix} 0.04583 & -0.00303 & 0.00085 & 0.00387 \\ 0.16532 & 0.12960 & -0.03385 & -0.08278 \\ 0.00976 & 0.00490 & 0.00011 & -0.00194 \end{bmatrix} \quad (12)$$

$$\beta_R = \begin{bmatrix} 0.05047 & -0.00160 & 0.01197 & 0.00162 \\ 0.07115 & 0.01197 & -0.03728 & 0.00372 \\ 0.00831 & 0.00162 & 0.00372 & 0.00007 \end{bmatrix} \quad (13)$$

Readings - Next Time

- * Cameron, A. C. and P. K. Trivedi. 2010. *Microeconometrics Using Stata* Revised Edition. Stata Press. (Chapters 3).
- * Morgan, M.S. 1992. *The History of Econometric Ideas*. Cambridge University Press. (Chapter 2: Measuring and Representing Business Cycles pp. 40 - 72).