# Expected Utility - Lecture IIa

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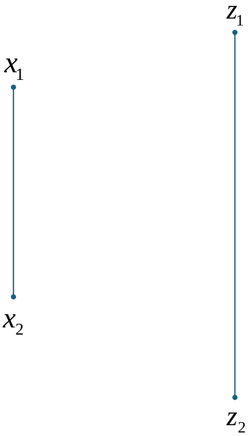
1. Defining the Lottery

2. Expected Utility

# Lotteries

- The first thing that we want to do is to define a lottery $L\left(x_1, x_2; P\right)$ where $x_1$ and $x_2$ are payoffs and $P$ is the probability that $x_1$ will occur.
- We assume that individuals can rank these lotteries:
  - Lottery $x$ is preferred to $z$ is denoted $L\left(x_1, x_2; P\right) \succ L\left(z_1, z_2; Q\right)$
  - Otherwise $z$ is indifferent or preferred to $x$ as denoted by $L\left(x_1, x_2, ; P\right) \preceq L\left(z_1, z_2; Q\right)$
- We are going to eventually transition to a utility function with the assumption that if $y_1 \succ y_2$ then $U\left(y_1\right) > U\left(y_2\right)$ - more is preferred to less.

# Outer Lottery



# Comparing two Lotteries

- As a starting point, we assume that we can choose a probability $Q_1^*$ such that $L\left(x_1, x_2; P\right) \approx L\left(z_1, z_2; Q_1^*\right)$.
- The basic notion is to use this relationship to compare two lotteries $L\left(x_1, x_2; P\right)$ and $L\left(x_1^*, x_2^*; p^*\right)$
- To do this we compute two comparisons on the index Lottery:
  - $L\left(x_1, x_2, ; P\right) \approx L\left(z_1, z_2; Q_1^*\right)$
  - $L\left(x_1^*, x_2^*; P^*\right) \approx L\left(z_1, z_2; Q_2^*\right)$
- If $Q_1^* > Q_2^*$ then $L\left(x_1, x_2; P\right) \succ L\left(x_1^*, x_2^*, P^*\right)$.

# Expected Utility

- Under this construction, we assume that we can construct a utility function (or mapping) $U(z)$ such that

$$\begin{gathered}\mathrm{E}\left[U(z)\right] = U(z_1)\,Q_1^* + U(z_2)\,(1 - Q_1^*) > \\ U(z_1)\,Q_2^* + U(z_2)\,(1 - Q_2^*)\end{gathered} \quad (1)$$