

AEB 7504 – Examination I, Fall 2025

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Please start each new question (1, 2, 3, 4 and the extra credit) on a new page of paper. Put your name and student identification number at the top of each page. Please use black pen on the answer pages and turn in any scrap pages. Remember that I need to be able to read your answer to give you credit. Your mission should you choose to accept it is to demonstrate your understanding to me. Turn your test in inside a white envelope with your name and student identification number on the outside of the envelope

1 Given the distribution function

$$f(x_1, x_2) = \frac{3}{112} \left(10 + \frac{1}{2}x_1 - x_1^2 - \frac{3}{10}x_2 - x_2^2 + \frac{1}{10}x_1x_2 \right) \quad (1)$$

where $x_1, x_2 \in [-1, 1]$.

- a Demonstrate that Equation 1 is a valid density function. [10 Points]
- b Compute the mean for both x_1 and x_2 . [10 Points]
- c Are x_1 and x_2 dependent or independent? [10 Points]

2 Starting with the probability density function

$$f(x) = 1 - \frac{x}{2} \quad (2)$$

defined over the interval $x \in [0, 2]$:

- a Derive the Moment Generating Function. [15 Points]

- b Use the Moment Generating Function to Derive the first and second non-central moments. [20 Points]

3 Assuming a probability density function of

$$g(x) = \frac{\exp(2-x)}{\exp(2)-1} \quad \ni: x \in [0, 2] \quad (3)$$

Derive the first, second and third non-central moments of this distribution [15 Points]. Compare these values with the numerical values from Table 1. Do you think that the values in Table 1 were drawn from this distribution [5 Points]? Hint: We can construct a Chi-Squared type of test

$$T = \left(\frac{\mu_x^{(1)} - m_x^{(1)}}{\mu_x^{(1)}} \right)^2 + \left(\frac{\mu_x^{(2)} - m_x^{(2)}}{\mu_x^{(2)}} \right)^2 + \left(\frac{\mu_x^{(3)} - m_x^{(3)}}{\mu_x^{(3)}} \right)^2 \sim \chi_{(3)}^2 \quad (4)$$

where $\mu_x^{(i)}$ is the i^{th} theoretical moment of the distribution and $m_x^{(i)}$ is the i^{th} empirical moment of the distribution.

- 4 I have been working with data on input prices paid for each state. Assuming that x_1 is the price of capital goods purchased in Florida and x_2 is the price of capital goods at the aggregate level.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim N \left[\begin{pmatrix} 0.6568 \\ 0.6221 \end{pmatrix}, \begin{pmatrix} 0.01940 & 0.01603 \\ 0.01603 & 0.01737 \end{pmatrix} \right] \quad (5)$$

- a What is the expected price for capital goods in Florida if the price of aggregate capital is 0.6350? [10 Points].
- b What is the conditional variance of capital price in Florida? [5 Points]

* **Extra Credit** Which justification of probabilities do you find most convincing and why? [5 Points]

Table 1: Observed Moments of the x Distribution

Obs	x	x^2	x^3	Obs	x	x^2	x^3
Obs	x	x^2	x^3	Obs	x	x^2	x^3
1	0.4934	0.2435	0.1201	26	1.3222	1.7482	2.3114
2	0.4171	0.1740	0.0726	27	0.3453	0.1193	0.0412
3	0.5211	0.2715	0.1415	28	0.0110	0.0001	0.0000
4	1.1951	1.4283	1.7070	29	0.0735	0.0054	0.0004
5	0.5717	0.3269	0.1869	30	0.5223	0.2728	0.1425
6	1.2607	1.5894	2.0039	31	0.1092	0.0119	0.0013
7	0.5609	0.3146	0.1765	32	0.2634	0.0694	0.0183
8	0.9070	0.8226	0.7461	33	0.4771	0.2276	0.1086
9	0.5711	0.3261	0.1862	34	0.3393	0.1152	0.0391
10	0.0236	0.0006	0.0000	35	0.1432	0.0205	0.0029
11	0.0868	0.0075	0.0007	36	0.1583	0.0251	0.0040
12	0.6642	0.4412	0.2930	37	0.1544	0.0238	0.0037
13	0.5375	0.2889	0.1553	38	0.5163	0.2666	0.1377
14	0.3812	0.1453	0.0554	39	0.3696	0.1366	0.0505
15	0.9130	0.8336	0.7611	40	0.3387	0.1147	0.0388
16	0.5293	0.2801	0.1482	41	0.1110	0.0123	0.0014
17	1.0443	1.0906	1.1390	42	0.6102	0.3724	0.2273
18	1.1286	1.2738	1.4377	43	0.6542	0.4280	0.2800
19	1.2705	1.6143	2.0510	44	0.7858	0.6174	0.4851
20	1.1082	1.2281	1.3610	45	0.4741	0.2247	0.1065
21	0.1493	0.0223	0.0033	46	0.6142	0.3773	0.2317
22	0.3245	0.1053	0.0342	47	1.4054	1.9752	2.7759
23	0.2236	0.0500	0.0112	48	0.3611	0.1304	0.0471
24	0.1424	0.0203	0.0029	49	0.1019	0.0104	0.0011
25	0.5335	0.2847	0.1519	50	1.3242	1.7536	2.3222
Sum					27.1449	22.2424	22.3250

Table 2: Chi-Squared Critical Values

Prob	1	2	3	4
0.99	6.6349	9.2103	11.3449	13.2767
0.95	3.8415	5.9915	7.8147	9.4877
0.90	2.7055	4.6052	6.2514	7.7794
0.50	0.4549	1.3863	2.3660	3.3567
0.10	0.0158	0.2107	0.5844	1.0636
0.05	0.0039	0.1026	0.3518	0.7107
0.01	0.0002	0.0201	0.1148	0.2971

Table 3: t and Normal Distributions

Degrees of Freedom	Probability			
	0.990	0.975	0.950	0.900
5	3.3649	2.5706	2.0150	1.4759
10	2.7638	2.2281	1.8125	1.3722
20	2.5280	2.0860	1.7247	1.3253
30	2.4573	2.0423	1.6973	1.3104
40	2.4233	2.0211	1.6839	1.3031
50	2.4033	2.0086	1.6759	1.2987
60	2.3901	2.0003	1.6706	1.2958
75	2.3771	1.9921	1.6654	1.2929
100	2.3642	1.9840	1.6602	1.2901
Infinity	2.3263	1.9600	1.6449	1.2816