

AEB 7504 – Examination I, Fall 2024

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Please start each new question (1, 2, 3, 4 and the extra credit) on a new page of paper. Put your name and student identification number at the top of each page. Please use black pen on the answer pages and turn in any scrap pages. Remember that I need to be able to read your answer to give you credit. Your mission should you choose to accept it is to demonstrate your understanding to me. Turn your test in inside a white envelope with your name and student identification number on the outside of the envelope

1 Given the distribution function

$$f(x_1, x_2) = \frac{1}{12} (3 - x_1 - x_2 + 3x_1x_2) \quad (1)$$

where $x_1, x_2 \in [-1, 1]$.

- a Demonstrate that Equation 1 is a valid density function. [10 Points]
- b Compute the mean for both x_1 and x_2 . [10 Points]
- c Are x_1 and x_2 dependent or independent? [10 Points]

2 Starting with the probability density function

$$f(x) = \frac{\exp(4 - x)}{\exp(4) - 1} \quad (2)$$

defined over the interval $x \in [0, 4]$:

- a Derive the Moment Generating Function. [15 Points]

- b Use the Moment Generating Function to Derive the first three non-central moments. [20 Points]

3 Assuming a probability density function of

$$g(x) = \frac{3(1-x^2)}{2}, \quad x \in [0, 1]. \quad (3)$$

Derive the first, second and third non-central moments of this distribution [15 Points]. Compare these values with the numerical values from Table 1. Do you think that the values in Table 1 were drawn from this distribution [5 Points]? Hint: We can construct a Chi-Squared type of test

$$T = \left(\frac{\mu_x^{(1)} - m_x^{(1)}}{\mu_x^{(1)}} \right)^2 + \left(\frac{\mu_x^{(2)} - m_x^{(2)}}{\mu_x^{(2)}} \right)^2 + \left(\frac{\mu_x^{(3)} - m_x^{(3)}}{\mu_x^{(3)}} \right)^2 \sim \chi_{(3)}^2 \quad (4)$$

where $\mu_x^{(i)}$ is the i^{th} theoretical moment of the distribution and $m_x^{(i)}$ is the i^{th} empirical moment of the distribution.

- 4 I have been working with data on the revenue generated by different farm sizes as measured by the Agricultural Resource Management Survey. Assume that x_1 is the sales of livestock measured 100 lbs. of 650 lb. feeder steers per farm and x_2 is the price of these in dollars per cwt. Based on my pooled data these variables are distributed

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim N \left[\begin{pmatrix} 56.388 \\ 126.407 \end{pmatrix}, \begin{pmatrix} 7.987 & -10.395 \\ -10.395 & 130.684 \end{pmatrix} \right] \quad (5)$$

- a What is the expected quantity of livestock produced if the price of feeder steers increases to \$150/cwt? [10 Points].
- b What is the conditional variance of the quantity of livestock produced? [5 Points]

* **Extra Credit** Which justification of probabilities do you find most convincing and why? [5 Points]

Table 1: Observed Moments of the x Distribution

Obs	x	x^2	x^3	Obs	x	x^2	x^3
1	0.27443	0.07531	0.02067	39	0.67135	0.45071	0.30259
2	0.19896	0.03959	0.00788	40	0.07558	0.00571	0.00043
3	0.01838	0.00034	0.00001	41	0.06272	0.00393	0.00025
4	0.44817	0.20085	0.09002	42	0.54508	0.29711	0.16195
5	0.77628	0.60261	0.46779	43	0.55135	0.30399	0.16760
6	0.40240	0.16192	0.06516	44	0.10577	0.01119	0.00118
7	0.78530	0.61670	0.48430	45	0.02609	0.00068	0.00002
8	0.28818	0.08305	0.02393	46	0.01513	0.00023	0.00000
9	0.03465	0.00120	0.00004	47	0.30871	0.09530	0.02942
10	0.47410	0.22477	0.10656	48	0.06527	0.00426	0.00028
11	0.19131	0.03660	0.00700	49	0.08170	0.00667	0.00055
12	0.45417	0.20627	0.09368	50	0.06229	0.00388	0.00024
13	0.55239	0.30513	0.16855	51	0.18434	0.03398	0.00626
14	0.33686	0.11347	0.03822	52	0.20249	0.04100	0.00830
15	0.17909	0.03207	0.00574	53	0.06653	0.00443	0.00029
16	0.11663	0.01360	0.00159	54	0.04210	0.00177	0.00007
17	0.63925	0.40865	0.26123	55	0.37570	0.14115	0.05303
18	0.56495	0.31917	0.18032	56	0.06303	0.00397	0.00025
19	0.07950	0.00632	0.00050	57	0.24282	0.05896	0.01432
20	0.46883	0.21980	0.10305	58	0.74981	0.56222	0.42156
21	0.75674	0.57266	0.43336	59	0.35436	0.12557	0.04450
22	0.21498	0.04621	0.00994	60	0.12781	0.01634	0.00209
23	0.10171	0.01034	0.00105	61	0.12222	0.01494	0.00183
24	0.72867	0.53096	0.38690	62	0.31646	0.10015	0.03169
25	0.59481	0.35380	0.21045	63	0.05543	0.00307	0.00017
26	0.28205	0.07955	0.02244	64	0.12555	0.01576	0.00198
27	0.45551	0.20749	0.09451	65	0.72418	0.52444	0.37979
28	0.47198	0.22276	0.10514	66	0.17792	0.03165	0.00563
29	0.82180	0.67535	0.55500	67	0.63913	0.40849	0.26107
30	0.25236	0.06368	0.01607	68	0.60585	0.36706	0.22238
31	0.01694	0.00029	0.00000	69	0.24400	0.05954	0.01453
32	0.68984	0.47587	0.32828	70	0.36856	0.13583	0.05006
33	0.27541	0.07585	0.02089	71	0.62073	0.38530	0.23917
34	0.03874	0.00150	0.00006	72	0.04660	0.00217	0.00010
35	0.06556	0.00430	0.00028	73	0.06437	0.00414	0.00027
36	0.17389	0.03024	0.00526	74	0.16757	0.02808	0.00471
37	0.03038	0.00092	0.00003	75	0.84764	0.71849	0.60902
38	0.32759	0.10732	0.03516 ³				
Sum					23.68902	12.09873	7.38863

Table 2: Chi-Squared Critical Values

Prob	1	2	3	4
0.99	6.6349	9.2103	11.3449	13.2767
0.95	3.8415	5.9915	7.8147	9.4877
0.90	2.7055	4.6052	6.2514	7.7794
0.50	0.4549	1.3863	2.3660	3.3567
0.10	0.0158	0.2107	0.5844	1.0636
0.05	0.0039	0.1026	0.3518	0.7107
0.01	0.0002	0.0201	0.1148	0.2971