

AEB 7504 - Examination 1

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Please start each new question (1, 2, 3, and the extra credit) on a new page of paper. Put your name and student identification number at the top of each page. Please use black pen on the answer pages and turn in any scrap pages. Remember that I need to be able to read your answer to give you credit. Your mission should you choose to accept it is to demonstrate your understanding to me. Turn your test in inside a white envelope with your name and student identification number on the outside of the envelope

1 Given the function

$$g(x_1, x_2) = \frac{9}{280} \left(\frac{10}{4} - \frac{x_1^2}{4} + \frac{2x_1x_2}{12} - \frac{x_2^2}{6} \right) \text{ for } -2 \leq x_1, x_2 \leq 2. \quad (1)$$

- a Demonstrate that $g(x_1, x_2)$ is a valid probability density function. [15 Points]
- b Derive the marginal distribution for x_1 . [15 Points]
- c Derive the covariance between x_1 and x_2 . [20 Points]

2 Given the function

$$f(x) = \frac{4 \exp(4)}{\exp(4) - 1} \exp(-4x) \text{ for } 0 \leq x \leq 1. \quad (2)$$

- a Derive the Moment Generating Function for $f(x)$. [10 Points]
- b Derive the first three moments of the distribution $f(x)$. [20 Points]

- c Compare the moments of the random variable in Table 1 with the theoretical moments from the moment generating function. Do you think that Equation 2 describes the empirical data? [5 Points]

3 Assume that x_1 and x_2 are distributed multivariate normal

$$N \left[\begin{pmatrix} 4.4321 \\ 7.0712 \end{pmatrix}, \begin{pmatrix} 1.3424 & 0.5116 \\ 0.5116 & 2.9013 \end{pmatrix} \right] \quad (3)$$

- a What is the expected value of x_2 given that $x_1 = 5.00$? [10 Points]
- b What is the conditional variance for x_2 ? [5 Points]

* **Extra Credit:** Describe the three ways we used to justify probability theory. [5 Points]

Table 1: Sample of Random Variables

Obs	x	x^2	x^3	x^4	Obs	x	x^2	x^3	x^4
1	0.26335	0.06936	0.01827	0.00481	41	0.26208	0.06868	0.01800	0.00472
2	0.81253	0.66021	0.53644	0.43587	42	0.18921	0.03580	0.00677	0.00128
3	0.10421	0.01086	0.00113	0.00012	43	0.09293	0.00864	0.00080	0.00007
4	0.01018	0.00010	0.00000	0.00000	44	0.50452	0.25454	0.12842	0.06479
5	0.17703	0.03134	0.00555	0.00098	45	0.30049	0.09030	0.02713	0.00815
6	0.38926	0.15152	0.05898	0.02296	46	0.18479	0.03415	0.00631	0.00117
7	0.31299	0.09797	0.03066	0.00960	47	0.13624	0.01856	0.00253	0.00034
8	0.18673	0.03487	0.00651	0.00122	48	0.00936	0.00009	0.00000	0.00000
9	0.15684	0.02460	0.00386	0.00061	49	0.23791	0.05660	0.01347	0.00320
10	0.14263	0.02034	0.00290	0.00041	50	0.01313	0.00017	0.00000	0.00000
11	0.22143	0.04903	0.01086	0.00240	51	0.07846	0.00616	0.00048	0.00004
12	0.01959	0.00038	0.00001	0.00000	52	0.10823	0.01171	0.00127	0.00014
13	0.02205	0.00049	0.00001	0.00000	53	0.34303	0.11767	0.04037	0.01385
14	0.08708	0.00758	0.00066	0.00006	54	0.69297	0.48021	0.33277	0.23060
15	0.01960	0.00038	0.00001	0.00000	55	0.16633	0.02767	0.00460	0.00077
16	0.27917	0.07794	0.02176	0.00607	56	0.12227	0.01495	0.00183	0.00022
17	0.01596	0.00025	0.00000	0.00000	57	0.13017	0.01695	0.00221	0.00029
18	0.02664	0.00071	0.00002	0.00000	58	0.12241	0.01498	0.00183	0.00022
19	0.05851	0.00342	0.00020	0.00001	59	0.12799	0.01638	0.00210	0.00027
20	0.04700	0.00221	0.00010	0.00000	60	0.17002	0.02891	0.00491	0.00084
21	0.00062	0.00000	0.00000	0.00000	61	0.11211	0.01257	0.00141	0.00016
22	0.19272	0.03714	0.00716	0.00138	62	0.37826	0.14308	0.05412	0.02047
23	0.29473	0.08687	0.02560	0.00755	63	0.05462	0.00298	0.00016	0.00001
24	0.11833	0.01400	0.00166	0.00020	64	0.01720	0.00030	0.00001	0.00000
25	0.22314	0.04979	0.01111	0.00248	65	0.48918	0.23930	0.11706	0.05726
26	0.39364	0.15495	0.06099	0.02401	66	0.22787	0.05192	0.01183	0.00270
27	0.11149	0.01243	0.00139	0.00015	67	0.25804	0.06659	0.01718	0.00443
28	0.26800	0.07182	0.01925	0.00516	68	0.03985	0.00159	0.00006	0.00000
29	0.14950	0.02235	0.00334	0.00050	69	0.26689	0.07123	0.01901	0.00507
30	0.03768	0.00142	0.00005	0.00000	70	0.11489	0.01320	0.00152	0.00017
31	0.25664	0.06586	0.01690	0.00434	71	0.33255	0.11059	0.03678	0.01223
32	0.04363	0.00190	0.00008	0.00000	72	0.02756	0.00076	0.00002	0.00000
33	0.07775	0.00605	0.00047	0.00004	73	0.24735	0.06118	0.01513	0.00374
34	0.07385	0.00545	0.00040	0.00003	74	0.03907	0.00153	0.00006	0.00000
35	0.26810	0.07188	0.01927	0.00517	75	0.14269	0.02036	0.00291	0.00041
36	0.50699	0.25704	0.13032	0.06607					
37	0.20036	0.04015	0.00804	0.00161	Sum	13.56587	4.27146	1.88041	1.04184
38	0.12740	0.01623	0.00207	0.00026					
39	0.10922	0.01193	0.00130	0.00014					
40	0.01860	0.00035	0.00001	0.00000					

Table 2: t and Normal Distribution

Obs	Probabilities									
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990		
10	-2.76377	-2.22814	-1.81246	-1.37218	1.37218	1.81246	2.22814	2.76377		
20	-2.52798	-2.08596	-1.72472	-1.32534	1.32534	1.72472	2.08596	2.52798		
25	-2.48511	-2.05954	-1.70814	-1.31635	1.31635	1.70814	2.05954	2.48511		
50	-2.40327	-2.00856	-1.67591	-1.29871	1.29871	1.67591	2.00856	2.40327		
75	-2.37710	-1.99210	-1.66543	-1.29294	1.29294	1.66543	1.99210	2.37710		
100	-2.36422	-1.98397	-1.66023	-1.29007	1.29007	1.66023	1.98397	2.36422		
150	-2.35146	-1.97591	-1.65508	-1.28722	1.28722	1.65508	1.97591	2.35146		
Normal	-2.32635	-1.95996	-1.64485	-1.28155	1.28155	1.64485	1.95996	2.32635		

Table 3: χ_n^2 Distribution					
Prob.	Degrees of Freedom				
	1	2	3	4	5
0.010	0.00016	0.02010	0.11483	0.29711	0.55430
0.025	0.00098	0.05064	0.21580	0.48442	0.83121
0.050	0.00393	0.10259	0.35185	0.71072	1.14548
0.100	0.01579	0.21072	0.58437	1.06362	1.61031
0.150	0.03577	0.32504	0.79777	1.36648	1.99382
0.250	0.10153	0.57536	1.21253	1.92256	2.67460
0.500	0.45494	1.38629	2.36597	3.35669	4.35146
0.750	1.32330	2.77259	4.10834	5.38527	6.62568
0.850	2.07225	3.79424	5.31705	6.74488	8.11520
0.900	2.70554	4.60517	6.25139	7.77944	9.23636
0.950	3.84146	5.99146	7.81473	9.48773	11.07050
0.975	5.02389	7.37776	9.34840	11.14329	12.83250
0.990	6.63490	9.21034	11.34487	13.27670	15.08627