

AEB 7504 Mathematical Statistics for Applied Econometrics – Midterm 1, 2022

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Please start each new question (1, 2, 3, 4, and the extra credit) on a new page of paper. Put your name and student identification number at the top of each page. Please use black pen on the answer pages and turn in any scrap pages. Remember that I need to be able to read your answer to give you credit. Turn your test in inside a white envelope with your name and student identification number on the outside of the envelop.

1 Starting with the function

$$f(x) = \frac{\exp\left(1 - \frac{1}{2}x\right)}{2\exp(1) - 2} \quad 0 \leq x \leq 2. \quad (1)$$

Hint make your life easier by collapsing several terms into a constant

$$\begin{aligned} f(x) &= k \exp\left(-\frac{x}{2}\right) \\ k &= \frac{\exp(1)}{2(\exp(1) - 1)} \end{aligned} \quad (2)$$

- a Demonstrate that $f(x)$ is a valid probability density function. [10 Points]
- b Derive the moment generating function for $f(x)$. [10 Points]
- c Use the moment generating function to derive the first and second moments. [10 Points]

- d Are these moments consistent with the empirical moments in Table 1 (Use the ‘test’)

$$T = \left(\left. \frac{dM_x(t)}{dt} \right|_{t \rightarrow 0} - \frac{1}{N} \sum_{i=1}^N x_i \right)^2 + \left(\left. \frac{dM_x(t)}{dt^2} \right|_{t \rightarrow 0} - \frac{1}{N} \sum_{i=1}^N x_i^2 \right)^2 \quad (3)$$

You may choose to use the statistics in Table 2 and 3). [5 Points]

2 Starting with the function

$$g(x_1, x_2) = \frac{3}{10} - \frac{9}{100}x_1^2 - \frac{3}{50}x_2^2 + \frac{3}{100}x_1x_2 \quad -1 \leq x_1, x_2 \leq 1. \quad (4)$$

Weierstrass’s Theorem: On a closed, bounded, and nonempty interval, a continuous function obtains an extreme point (i.e., maximum or minimum). This extreme point will occur at either a local extreme point (i.e., local maximum or local minimum) or a boundary point. [Hint – I think that this will be a useful statement for this question].

- a Prove that $g(x_1, x_2)$ is a valid probability density function. [10 Points]
- b Derive the marginal density function for x_2 . [10 Points]
- c Derive the conditional distribution function for x_1 given $x_2 = 1/2$. Are x_1 and x_2 independent? [10 Points]
- d Derive the covariance between x_1 and x_2 . [10 Points]

3 Assuming that x and y are distributed joint normal

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim N \left[\begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 & -2 \\ -2 & 6 \end{pmatrix} \right] \quad (5)$$

- a Derive the conditional expectation of x based on y . [10 Points]
- b Derive the conditional variance of x based on y . [10 Points]

4 Assume that $f(x) = 2 - x$ is the probability functions for $x \in [0, 2]$. Construct the probability density function for $y = x^2$. [5 Points]

* **Extra Credit:** Describe Popper’s falsifiability requirement [10 Points].

Table 1: Sample Data for Question 1

Obs	x	x^2	Obs	x	x^2
1	1.4894	2.2184	26	0.0216	0.0005
2	1.1458	1.3129	27	0.0347	0.0012
3	1.3697	1.8762	28	0.5908	0.3491
4	0.7528	0.5667	29	1.0694	1.1437
5	1.0750	1.1555	30	0.2348	0.0551
6	1.6936	2.8683	31	0.7658	0.5864
7	0.1946	0.0379	32	1.1778	1.3873
8	1.7193	2.9560	33	1.7663	3.1197
9	0.3829	0.1466	34	1.3856	1.9200
10	1.0663	1.1370	35	1.1501	1.3227
11	1.1380	1.2951	36	0.9086	0.8255
12	0.1811	0.0328	37	0.0078	0.0001
13	0.0781	0.0061	38	0.4946	0.2447
14	0.9807	0.9617	39	0.8472	0.7177
15	1.8180	3.3049	40	0.2212	0.0489
16	1.3497	1.8216	41	0.7168	0.5138
17	1.4384	2.0691	42	0.2749	0.0756
18	0.8802	0.7747	43	0.9457	0.8943
19	0.1438	0.0207	44	0.0631	0.0040
20	1.2719	1.6177	45	1.5821	2.5030
21	0.2538	0.0644	46	0.6838	0.4675
22	1.7117	2.9300	47	0.4624	0.2138
23	0.9577	0.9171	48	0.8737	0.7634
24	1.4913	2.2240	49	0.1491	0.0222
25	0.8002	0.6404	50	0.6298	0.3966
			Sum	42.4418	50.5328

Table 2: Numeric Values for the Normal Distribution

x	$f(x)$	$F(x)$	x	$f(x)$	$F(x)$
-3.25	0.0020	0.0006	0.00	0.3989	0.5000
-3.00	0.0044	0.0013	0.25	0.3867	0.5987
-2.75	0.0091	0.0030	0.50	0.3521	0.6915
-2.50	0.0175	0.0062	0.75	0.3011	0.7734
-2.25	0.0317	0.0122	1.00	0.2420	0.8413
-2.00	0.0540	0.0228	1.25	0.1826	0.8944
-1.75	0.0863	0.0401	1.50	0.1295	0.9332
-1.50	0.1295	0.0668	1.75	0.0863	0.9599
-1.25	0.1826	0.1056	2.00	0.0540	0.9772
-1.00	0.2420	0.1587	2.25	0.0317	0.9878
-0.75	0.3011	0.2266	2.50	0.0175	0.9938
-0.50	0.3521	0.3085	2.75	0.0091	0.9970
-0.25	0.3867	0.4013	3.00	0.0044	0.9987
0.00	0.3989	0.5000	3.25	0.0020	0.9994

Table 3: Numeric Values for the Chi-Square Distribution

Probability	Degrees of Freedom			
	1	2	3	4
0.025	0.0010	0.0506	0.2158	0.4844
0.050	0.0039	0.1026	0.3518	0.7107
0.100	0.0158	0.2107	0.5844	1.0636
0.900	2.7055	4.6052	6.2514	7.7794
0.950	3.8415	5.9915	7.8147	9.4877
0.975	5.0239	7.3778	9.3484	11.1433