

AEB 7571 Econometric Methods I – First Midterm

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September 27, 2019

1 Defining the function

$$f(x, y) = \frac{9}{140} (4 + x + 2y - x^2 y^2), \quad x \in [-1, 1] \quad y \in [-1, 1] \quad (1)$$

- a Is this function a valid probability density function? (Hint: Evaluate the hessian at $x = -1/2$ and $y = -1/2$ if the Hessian is indefinite, it has no local minimum or maximum) [15 Points]
- b Derive the marginal density functions for x and y . [15 Points]
- c Are random variables independent? (Hint: Don't spend too much time here - what would be required? Does that look feasible?) [10 Points]

2 Assuming the distribution function

$$g(x) = k \exp\left(\frac{x}{2}\right) \text{ where } k = \frac{1}{2 \exp(1) - 2}, \quad x \in [0, 2] \quad (2)$$

- a Derive the moment generating function for x . [15 Points]
- b Compute the first and second moments of the distribution function using the moment generating function. [15 Points]
- c Using the test

$$T = (\mu_1 - m_1)^2 + (\mu_2 - m_2)^2 \sim \chi_{(2)}^2 \quad (3)$$

Do you think that the distribution in Table 1 was drawn from the distribution of x (given the distribution function $g(x)$)? See the statistical tables given in Tables 2 and 3 [10 Points]

3 Given that x_1 and x_2 are distributed multivariate normal

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim N \left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix} \right) \quad (4)$$

1 What is the expected value of x_2 given $x_1 = 0$? [10 Points]

2 What is the variance of x_2 given $x_1 = 0$? [10 Points]

* Extra Credit: What are three ways to justify probability theory [3 points for each correct answer]?

Table 1: Sample of Random Draws – Question 2

Obs	x	x^2	x^3	Obs	x	x^2	x^3
1	1.9413	3.7685	7.3157	26	0.7015	0.4921	0.3452
2	1.1696	1.3679	1.5998	27	0.4960	0.2460	0.1220
3	0.7264	0.5277	0.3834	28	0.1528	0.0233	0.0036
4	1.9250	3.7057	7.1336	29	0.8572	0.7347	0.6298
5	1.1948	1.4276	1.7058	30	1.0388	1.0792	1.1211
6	0.8346	0.6966	0.5813	31	1.9617	3.8482	7.5490
7	1.4448	2.0873	3.0157	32	0.7001	0.4901	0.3431
8	0.9941	0.9883	0.9825	33	0.7776	0.6047	0.4703
9	1.3040	1.7005	2.2175	34	1.3578	1.8437	2.5034
10	0.9503	0.9031	0.8582	35	0.7264	0.5277	0.3833
11	0.5400	0.2916	0.1575	36	1.8546	3.4396	6.3792
12	0.8430	0.7106	0.5990	37	1.5275	2.3332	3.5638
13	1.2282	1.5086	1.8529	38	1.6955	2.8748	4.8744
14	1.4515	2.1070	3.0584	39	1.8454	3.4055	6.2846
15	1.0613	1.1264	1.1955	40	1.4309	2.0475	2.9297
16	0.7070	0.4998	0.3534	41	0.9810	0.9624	0.9442
17	1.5858	2.5148	3.9880	42	1.5910	2.5314	4.0276
18	0.3058	0.0935	0.0286	43	0.8671	0.7518	0.6518
19	0.2275	0.0518	0.0118	44	0.8047	0.6476	0.5211
20	0.7809	0.6098	0.4762	45	0.3460	0.1197	0.0414
21	0.2268	0.0515	0.0117	46	0.8913	0.7944	0.7080
22	1.3976	1.9533	2.7299	47	1.9911	3.9643	7.8932
23	1.7383	3.0218	5.2529	48	1.8487	3.4177	6.3184
24	0.0276	0.0008	0.0000	49	0.1863	0.0347	0.0065
25	1.6853	2.8404	4.7870	50	0.6417	0.4118	0.2642

Table 2: Numeric Values for the Normal Distribution

x	$f(x)$	$F(x)$	x	$f(x)$	$F(x)$
-3.25	0.0020	0.0006	0.00	0.3989	0.5000
-3.00	0.0044	0.0013	0.25	0.3867	0.5987
-2.75	0.0091	0.0030	0.50	0.3521	0.6915
-2.50	0.0175	0.0062	0.75	0.3011	0.7734
-2.25	0.0317	0.0122	1.00	0.2420	0.8413
-2.00	0.0540	0.0228	1.25	0.1826	0.8944
-1.75	0.0863	0.0401	1.50	0.1295	0.9332
-1.50	0.1295	0.0668	1.75	0.0863	0.9599
-1.25	0.1826	0.1056	2.00	0.0540	0.9772
-1.00	0.2420	0.1587	2.25	0.0317	0.9878
-0.75	0.3011	0.2266	2.50	0.0175	0.9938
-0.50	0.3521	0.3085	2.75	0.0091	0.9970
-0.25	0.3867	0.4013	3.00	0.0044	0.9987
0.00	0.3989	0.5000	3.25	0.0020	0.9994

Table 3: Numeric Values for the Chi-Square Distribution

Probability	Degrees of Freedom			
	1	2	3	4
0.025	0.0010	0.0506	0.2158	0.4844
0.050	0.0039	0.1026	0.3518	0.7107
0.100	0.0158	0.2107	0.5844	1.0636
0.900	2.7055	4.6052	6.2514	7.7794
0.950	3.8415	5.9915	7.8147	9.4877
0.975	5.0239	7.3778	9.3484	11.1433