

AEB 7571 - Midterm 1 - 2018

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September 27, 2018

1. Using the function

$$f(x, y) = \frac{4}{81} (9 - 3x - 3y + xy) \quad x \in [0, 3], y \in [0, 3] \quad (1)$$

- a. Is this function a valid distribution function? [10 Points]
- b. Derive the marginal distribution of x and y . [10 Points]
- c. What is the covariance between x and y . [10 Points]

2. Starting the function

$$f(x) = \frac{3}{2} - x, x \in (0, 1) \quad (2)$$

- a. Derive the moment generating function for x . [15 Points]
- b. Derive the first and second moments of x using the moment generating function. [15 Points]
- c. Are the results in Table consistent with the first two moments of this distribution? You can use

$$T = (\hat{m}_1 - \mu_1)^2 + (\hat{m}_2 - \mu_2)^2 \sim \chi_2^2. \quad (3)$$

[5 Points]

3. Given the bivariate normal distribution

$$\begin{bmatrix} x \\ y \end{bmatrix} \sim N \left[\begin{pmatrix} 77.5142 \\ 65.025 \end{pmatrix}, \begin{bmatrix} 1.262 & 76.771 \\ 76.771 & 9952.651 \end{bmatrix} \right] \quad (4)$$

- a. Derive the conditional expectation of y given that $x = 80.00$. [10 Points].
- b. Derive the conditional variance for y given that $x = 80.00$. [10 Points].

4. Starting with the distribution function

$$f(x) = 2(1 - x), x \in [0, 1] \quad (5)$$

and $y = \phi(x) = x^2$. Derive the probability density function for y . [15 Points]

* *Extra Credit* What are the three approaches to justifying probability? [12 Points - 4 for each].

Table 1: Observed x for Question 2

Obs.	x	x^2
1	0.1415	0.0200
2	0.5947	0.3537
3	0.7133	0.5088
4	0.5786	0.3348
5	0.1417	0.0201
6	0.8707	0.7581
7	0.0093	0.0001
8	0.0562	0.0032
9	0.4010	0.1608
10	0.1656	0.0274
11	0.8561	0.7330
12	0.0146	0.0002
13	0.2644	0.0699
14	0.3869	0.1497
15	0.2924	0.0855
16	0.4245	0.1802
17	0.6067	0.3680
18	0.6161	0.3796
19	0.4107	0.1687
20	0.2224	0.0495
21	0.1697	0.0288
22	0.8409	0.7071
23	0.5757	0.3315
24	0.9903	0.9806
25	0.5379	0.2894
Sum	10.8819	6.7086
Sample Average	0.4353	0.0116

Table 2: Numeric Values for the Normal Distribution

x	$f(x)$	$F(x)$	x	$f(x)$	$F(x)$
-3.25	0.0020	0.0006	0.00	0.3989	0.5000
-3.00	0.0044	0.0013	0.25	0.3867	0.5987
-2.75	0.0091	0.0030	0.50	0.3521	0.6915
-2.50	0.0175	0.0062	0.75	0.3011	0.7734
-2.25	0.0317	0.0122	1.00	0.2420	0.8413
-2.00	0.0540	0.0228	1.25	0.1826	0.8944
-1.75	0.0863	0.0401	1.50	0.1295	0.9332
-1.50	0.1295	0.0668	1.75	0.0863	0.9599
-1.25	0.1826	0.1056	2.00	0.0540	0.9772
-1.00	0.2420	0.1587	2.25	0.0317	0.9878
-0.75	0.3011	0.2266	2.50	0.0175	0.9938
-0.50	0.3521	0.3085	2.75	0.0091	0.9970
-0.25	0.3867	0.4013	3.00	0.0044	0.9987
0.00	0.3989	0.5000	3.25	0.0020	0.9994

Table 3: Numeric Values for the Chi-Square Distribution

Probability	Degrees of Freedom			
	1	2	3	4
0.025	0.0010	0.0506	0.2158	0.4844
0.050	0.0039	0.1026	0.3518	0.7107
0.100	0.0158	0.2107	0.5844	1.0636
0.900	2.7055	4.6052	6.2514	7.7794
0.950	3.8415	5.9915	7.8147	9.4877
0.975	5.0239	7.3778	9.3484	11.1433