

AEB 7571 - Midterm 1 - 2017

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September 25, 2017

1. Using the function

$$f(x, y) = \frac{1}{36} (3 - x) (4 - y) \quad x \in [0, 3], y \in [0, 4] \quad (1)$$

- Is this function a valid distribution function? [10 Points]
- Derive the marginal distribution of x and y . [10 Points]
- What is the covariance between x and y . [10 Points]

2. Starting the function

$$f(x) = \frac{\exp\left(-\frac{3}{2}\right)}{2} \exp\left(\frac{3-x}{2}\right), x \in (0, \infty) \quad (2)$$

- Derive the moment generating function for x . [15 Points]
- Derive the first and second moments of x using the moment generating function. [15 Points]
- Are the results in Table consistent with the first two moments of this distribution? You can use

$$T = (\hat{m}_1 - \mu_1)^2 + (\hat{m}_2 - \mu_2)^2 \sim \chi_2^2. \quad (3)$$

[5 Points]

3. Given the bivariate normal distribution

$$\begin{bmatrix} x \\ y \end{bmatrix} \sim N \left[\begin{pmatrix} 1.35358 \\ 1.172347 \end{pmatrix}, \begin{bmatrix} 0.57232 & -0.22184 \\ -0.22184 & 1.17236 \end{bmatrix} \right] \quad (4)$$

- a. Derive the conditional expectation of x given that $y = 0.50$. [10 Points].
- b. Derive the conditional variance for x given that $y = 0.50$. [10 Points].

4. Starting with the distribution function

$$f(x) = 2x, x \in [0, 1] \quad (5)$$

and $y = \phi(x) = x^{1/2}$. Derive the probability density function for y . [15 Points]

* *Extra Credit* What are the three approaches to justifying probability? [12 Points - 4 for each].

Table 1: Observed x for Question 2

Obs.	x	x^2
1	3.0430	9.2596
2	5.1533	26.5569
3	0.5851	0.3423
4	0.0459	0.0021
5	1.3481	1.8173
6	2.2816	5.2056
7	1.2723	1.6188
8	5.1026	26.0368
9	4.1352	17.0996
10	3.2368	10.4772
11	0.1384	0.0191
12	2.3134	5.3516
13	0.2515	0.0633
14	4.3604	19.0129
15	1.5036	2.2607
16	1.1159	1.2452
17	0.5186	0.2689
18	1.1629	1.3523
19	3.8502	14.8241
20	3.9597	15.6789
21	0.9194	0.8453
22	1.4942	2.2327
23	0.7400	0.5476
24	5.4870	30.1072
25	0.8819	0.7777
26	2.0050	4.0202
27	2.5640	6.5742
28	1.9820	3.9281
29	0.0208	0.0004
30	2.1978	4.8305
31	3.6422	13.2654
32	3.4340	11.7926
33	0.0232	0.0005
34	1.0640	1.1321
35	0.1435	0.0206
	2.0565	6.8162

Table 2: Numeric Values for the Normal Distribution

x	$f(x)$	$F(x)$	x	$f(x)$	$F(x)$
-3.25	0.0020	0.0006	0.00	0.3989	0.5000
-3.00	0.0044	0.0013	0.25	0.3867	0.5987
-2.75	0.0091	0.0030	0.50	0.3521	0.6915
-2.50	0.0175	0.0062	0.75	0.3011	0.7734
-2.25	0.0317	0.0122	1.00	0.2420	0.8413
-2.00	0.0540	0.0228	1.25	0.1826	0.8944
-1.75	0.0863	0.0401	1.50	0.1295	0.9332
-1.50	0.1295	0.0668	1.75	0.0863	0.9599
-1.25	0.1826	0.1056	2.00	0.0540	0.9772
-1.00	0.2420	0.1587	2.25	0.0317	0.9878
-0.75	0.3011	0.2266	2.50	0.0175	0.9938
-0.50	0.3521	0.3085	2.75	0.0091	0.9970
-0.25	0.3867	0.4013	3.00	0.0044	0.9987
0.00	0.3989	0.5000	3.25	0.0020	0.9994

Table 3: Numeric Values for the Chi-Square Distribution

Probability	Degrees of Freedom			
	1	2	3	4
0.025	0.0010	0.0506	0.2158	0.4844
0.050	0.0039	0.1026	0.3518	0.7107
0.100	0.0158	0.2107	0.5844	1.0636
0.900	2.7055	4.6052	6.2514	7.7794
0.950	3.8415	5.9915	7.8147	9.4877
0.975	5.0239	7.3778	9.3484	11.1433