

AEB 7571 - Econometric Methods I

First Midterm

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1. Demonstrate that the function

$$f(x, y) = \frac{4}{9}xy + \frac{8}{9}x + \frac{8}{9}y, \quad x, y \in [0, 1] \quad (1)$$

is a valid distribution function. [15 Points]

2. Assuming that we start with the distribution

$$f(x) = \frac{1}{2}x, \quad x \in [0, 2] \quad (2)$$

- a. Is this a valid probability density function? [5 Points]
- b. Derive the population (theoretical) mean of x . [10 Points]
- c. Using the transformation $y = \ln(x)$ derive the distribution function for y . [10 Points]
- d. Is this distribution function valid? (Focus on integrating to one) [5 Points]

3. Moment Generating Functions

- a. Derive the Moment Generating Function of

$$f(x) = \frac{1}{2}x, \quad x \in [0, 2] \quad (3)$$

[10 Points]

b. Using the Moment Generating Function

$$M(t) = \frac{0.790989 - 0.290989e^{2t}}{0.5 - t} \quad (4)$$

- i. Derive the first, second, and third moments. [5 Points each - 15 Points total]
- ii. Given the data in Table 1, test for whether the data could have been drawn from this population function. Hint: Use

$$T = (m_1 - \mu_1)^2 + (m_2 - \mu_2)^2 + (m_3 - \mu_3)^2 \sim \chi_{(3)}^2. \quad (5)$$

[10 Points]

4. Assuming the bivariate normal distribution

$$\begin{bmatrix} x \\ y \end{bmatrix} \sim N \left(\begin{bmatrix} 10.0 \\ -2.5 \end{bmatrix}, \begin{bmatrix} 22.5625 & -29.5313 \\ -29.5313 & 56.2500 \end{bmatrix} \right) \quad (6)$$

- a. Derive the conditional distribution of Y given X . [10 Points]
- b. Compute the conditional variance of Y given X . [10 Points]

* *Extra Credit* What are the three approaches to justifying probability? [12 Points - 4 for each].

Table 1: Random Variables for Moment Generating Function

Obs	x	x^2	x^3
1	0.7690	0.5913	0.4547
2	1.7044	2.9050	4.9512
3	0.2938	0.0863	0.0254
4	0.7671	0.5884	0.4514
5	0.1061	0.0113	0.0012
6	0.9380	0.8799	0.8253
7	0.5260	0.2766	0.1455
8	0.9865	0.9731	0.9600
9	1.0136	1.0274	1.0415
10	1.3627	1.8569	2.5304
11	1.3703	1.8778	2.5731
12	0.7927	0.6284	0.4982
13	0.8738	0.7636	0.6672
14	0.1280	0.0164	0.0021
15	1.0850	1.1773	1.2774
16	1.4360	2.0621	2.9612
17	1.6866	2.8448	4.7981
18	0.3671	0.1348	0.0495
19	1.7595	3.0959	5.4474
20	1.1156	1.2446	1.3885
21	0.8018	0.6429	0.5155
22	1.3744	1.8890	2.5963
23	1.1484	1.3189	1.5146
24	1.1758	1.3824	1.6254
25	0.9184	0.8434	0.7746
Sum/25	0.9800	1.1647	1.5230

Table 2: Numeric Values for the Normal Distribution

x	$f(x)$	$F(x)$	x	$f(x)$	$F(x)$
-3.25	0.0020	0.0006	0.00	0.3989	0.5000
-3.00	0.0044	0.0013	0.25	0.3867	0.5987
-2.75	0.0091	0.0030	0.50	0.3521	0.6915
-2.50	0.0175	0.0062	0.75	0.3011	0.7734
-2.25	0.0317	0.0122	1.00	0.2420	0.8413
-2.00	0.0540	0.0228	1.25	0.1826	0.8944
-1.75	0.0863	0.0401	1.50	0.1295	0.9332
-1.50	0.1295	0.0668	1.75	0.0863	0.9599
-1.25	0.1826	0.1056	2.00	0.0540	0.9772
-1.00	0.2420	0.1587	2.25	0.0317	0.9878
-0.75	0.3011	0.2266	2.50	0.0175	0.9938
-0.50	0.3521	0.3085	2.75	0.0091	0.9970
-0.25	0.3867	0.4013	3.00	0.0044	0.9987
0.00	0.3989	0.5000	3.25	0.0020	0.9994

Table 3: Numeric Values for the Chi-Square Distribution

Probability	Degrees of Freedom			
	1	2	3	4
0.025	0.0010	0.0506	0.2158	0.4844
0.050	0.0039	0.1026	0.3518	0.7107
0.100	0.0158	0.2107	0.5844	1.0636
0.900	2.7055	4.6052	6.2514	7.7794
0.950	3.8415	5.9915	7.8147	9.4877
0.975	5.0239	7.3778	9.3484	11.1433