

AEB 7571 First Midterm Examination

Charles B. Moss

September 27, 2015

1. Given the function

$$f(x_1, x_2) = 2 - x_1 - x_2, x_1, x_2 \in [0, 1]. \quad (1)$$

- a. Is $f(x_1, x_2)$ a valid probability density function? [10 Points]
- b. Derive the marginal probability density functions for x_1 and x_2 . [10 Points]
- c. Are x_1 and x_2 independent? [5 Points]

2. Starting with the probability density function

$$f(x) = \frac{4}{3} - \frac{2}{3}x, x \in [0, 1] \quad (2)$$

Given the transformation

$$y = g(x) = \frac{1}{x^2} \quad (3)$$

- a. What properties of $g(x)$ are required for a transformation of variables approach defining a new distribution function? [5 Points]
- b. Derive the new distribution function (note the range of the new random variable). [10 Points]

3. Moment Generating Functions:

- a. Derive the moment generating function for the probability density function

$$f(x) = 1 - \frac{x}{2}, x \in [0, 2] \quad (4)$$

[10 Points]

- b. Using the moment generating function for the Laplace distribution

$$M(t) = \frac{\exp(\mu t)}{(1 - b^2 t^2)}. \quad (5)$$

- i. Derive the first three moments of the distribution. [10 Points]
- ii. Are these moments consistent with the data presented in Table 1? [15 Points]

4. Assume that two variables x_1 and x_2 are distributed normal

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim N \left[\begin{pmatrix} 5 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 & -1 \\ -1 & 3 \end{pmatrix} \right] \quad (6)$$

Table 1: Sample of "Laplace" Variables

Observation	x	x^2	x^3
1	-26.817	719.167	-19,286.110
2	-16.656	277.420	-4,620.690
3	6.053	36.640	221.784
4	5.144	26.464	136.142
5	4.146	17.193	71.290
6	-2.113	4.465	-9.435
7	3.743	14.006	52.419
8	-4.396	19.326	-84.961
9	4.400	19.363	85.206
10	-0.421	0.177	-0.075
11	2.463	6.069	14.950
12	3.954	15.633	61.811
13	13.217	174.684	2,308.774
14	21.985	483.346	10,626.420
15	3.593	12.909	46.378
16	5.036	25.359	127.700
17	5.330	28.413	151.449
18	-1.182	1.397	-1.651
19	2.087	4.357	9.094
20	-20.361	414.567	-8,440.967
21	2.883	8.314	23.973
22	36.268	1,315.393	47,707.139
23	16.191	262.148	4,244.423
24	-0.463	0.215	-0.099
25	-6.351	40.335	-256.166
Sum	57.735	3,927.359	33,188.798
Sum/25	2.309	157.094	1,327.552

b. What is the mean of x_2 given that $x_1 = 4$? [10 Points]

a. What is the mean and variance of the distribution for x_1 conditional on x_2 ? [15 Points]

* **Extra Credit:** Given the first three moments of the distribution is it possible to differentiate between the Laplace distribution and the normal distribution. Remember that the moment generating function for the normal distribution function is

$$M(t) = \exp \left[\mu t + \frac{1}{2} \sigma^2 t^2 \right]. \quad (7)$$

[10 Points]