

First Midterm Examination

AEB 6571 Econometric Methods I

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1. Demonstrate that the function

$$g(x, y) = \frac{3}{8} - \frac{3x^2}{16} + \frac{3xy}{64} - \frac{3y^2}{16} \quad (1)$$

as depicted in Figure 1 is a valid probability density function for $x \in [-1, 1]$, $y \in [-1, 1]$. *25 points*

2. Compute the conditional mean of y given that $x = 0$ using the density function from Equation 1. *10 points*
3. Given the probability density function in Equation 1 are x and y independent? *15 points*
4. Given that $f(x) = 1/2$ for $x \in [0, 2]$ and $y = g(x) = \ln(x)$ derive the distribution of y . *10 points*
5. Derive the moment generating function for

$$f(x) = 3 \exp(-3x). \quad (2)$$

Use this to compute the first and second (noncentral moments) of the distribution. Compare the theoretical moments of the exponential distribution with the sample moments for the data in Table 1. Are the moments the same? *25 points*

Table 1: Sample Data for Exponential Distribution

Obs	x	x^2	x^3
1	0.2842	0.0808	0.0230
2	0.2357	0.0556	0.0131
3	0.0806	0.0065	0.0005
4	0.9729	0.9465	0.9208
5	0.0658	0.0043	0.0003
6	0.5058	0.2559	0.1294
7	1.0058	1.0116	1.0174
8	0.3832	0.1468	0.0563
9	0.3491	0.1219	0.0425
10	0.1179	0.0139	0.0016
11	0.2461	0.0606	0.0149
12	0.3902	0.1523	0.0594
13	1.3659	1.8657	2.5484
14	0.5357	0.2870	0.1537
15	0.0241	0.0006	0.0000
16	0.0648	0.0042	0.0003
17	0.0169	0.0003	0.0000
18	0.0041	0.0000	0.0000
19	0.0568	0.0032	0.0002
20	0.3976	0.1581	0.0629
21	0.2335	0.0545	0.0127
22	0.1710	0.0293	0.0050
23	0.2504	0.0627	0.0157
24	0.1785	0.0319	0.0057
25	0.8666	0.7509	0.6507
26	0.0518	0.0027	0.0001
27	0.0368	0.0014	0.0000
28	0.0769	0.0059	0.0005
29	0.2483	0.0617	0.0153
30	0.2287	0.0523	0.0120
Sum	9.4459	6.2288	5.7625

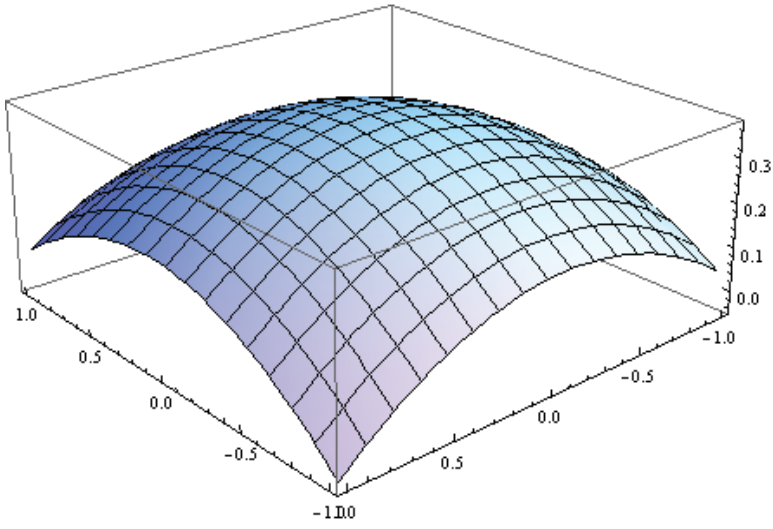


Figure 1: Probability Density Function

Table 2: Standard Normal Probabilities

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0.00	0.5000	0.55	0.7088	1.10	0.8643	1.65	0.9505
0.05	0.5199	0.60	0.7257	1.15	0.8749	1.70	0.9554
0.10	0.5398	0.65	0.7422	1.20	0.8849	1.75	0.9599
0.15	0.5596	0.70	0.7580	1.25	0.8944	1.80	0.9641
0.20	0.5793	0.75	0.7734	1.30	0.9032	1.85	0.9678
0.25	0.5987	0.80	0.7881	1.35	0.9115	1.90	0.9713
0.30	0.6179	0.85	0.8023	1.40	0.9192	1.95	0.9744
0.35	0.6368	0.90	0.8159	1.45	0.9265	2.00	0.9772
0.40	0.6554	0.95	0.8289	1.50	0.9332	2.05	0.9798
0.45	0.6736	1.00	0.8413	1.55	0.9394	2.10	0.9821
0.50	0.8413	1.05	0.8531	1.60	0.9452	2.15	0.9842

Table 3: Chi-squared Probability Table

Degrees of Freedom	0.99	0.98	0.90	0.80	0.70	0.50	
1	0.000	0.001	0.004	0.016	0.064	0.148	0.455
2	0.020	0.040	0.103	0.211	0.446	0.713	1.386
3	0.115	0.185	0.352	0.584	1.005	1.424	2.366
4	0.297	0.429	0.711	1.064	1.649	2.195	3.357
5	0.554	0.752	1.145	1.610	2.343	3.000	4.351
	0.30	0.20	0.10	0.05	0.02	0.01	0.001
1	1.074	1.642	2.706	3.841	5.412	6.635	10.827
2	2.408	3.219	4.605	5.991	7.824	9.210	13.815
3	3.665	4.642	6.251	7.814	9.837	11.345	16.268
4	4.878	5.989	7.779	9.488	11.668	13.277	18.465
5	6.064	7.289	9.236	11.070	13.388	15.086	20.517

6. Assuming a Binomial distribution what is the probability of drawing $n \leq 3$ out of $N = 10$ draws given $p = 0.40$? Using the normal distribution table, what is the probability assuming normality? *15 points*