

Lecture XXXVIII: Numerical Approximation of Production Functions

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- 1 The CES Production Function
- 2 Approximation of Supply and Demand Curves

The CES Production Function

- Suppose that we needed a CES production function for a computable general equilibrium model.
 - We could collect data and estimate the function.
 - We could approximate the function based on optimizing behavior.

- Taking the latter approach, we start with the solution from Lecture 29

$$y(x_1, x_2, x_3, a_1, a_2, a_3, s) = \left[a_1^{1/s} x_1^{(s-1)/s} + a_2^{1/s} x_2^{(s-1)/s} + a_3^{1/s} x_3^{(s-1)/s} \right]^{s/(s-1)}$$

$$x_i(x_1, x_2, x_3, y, a_1, a_2, a_3, s) = \frac{y \alpha_i}{w_i^s \left[\sum_{i=1}^3 a_i w_i^{1-s} \right]^{s/(s-1)}} \quad (1)$$

- Assume that we have a single data point – for example, Table 5 presents the data for corn production in Georgia in 2010.

Corn Production for Georgia in 2010

Factor	Level	Price
Nitrogen	177 lbs.	\$ 398/ton
Phosphate	68 lbs.	\$ 507/ton
Potash	99 lbs.	\$ 511/ton
Corn	145 bu.	\$ 5.95/bu.

- The concept is that taking the level of prices and inputs in Equation 1 as a given, we can solve for the parameters of the production function. For example, choose the levels of parameters to minimize the error of approximation

$$\begin{aligned}
 & \min_{a_1, a_2, a_3, s} \left(y - \left[a_1^{1/s} x_1^{(s-1)/s} + a_2^{1/s} x_2^{(s-1)/s} + a_3^{1/s} x_3^{(s-1)/s} \right]^{s/(s-1)} \right)^2 + \\
 & \left(x_1 - \frac{y\alpha_1}{w_1^s \left[\sum_{i=1}^3 a_i w_i^{1-s} \right]^{s/(s-1)}} \right)^2 + \left(x_2 - \frac{y\alpha_2}{w_2^s \left[\sum_{i=1}^3 a_i w_i^{1-s} \right]^{s/(s-1)}} \right)^2 + \\
 & \left(x_3 - \frac{y\alpha_3}{w_3^s \left[\sum_{i=1}^3 a_i w_i^{1-s} \right]^{s/(s-1)}} \right)^2 +
 \end{aligned}
 \tag{2}$$

- While, I have used the CES function, this approach can be used as long as the number of parameters equals the number of equations (production function plus the number of demands). For example, the same approach can be used for the Cobb-Douglas function

$$y = \alpha_0 x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3} \quad (3)$$

Approximation of Supply and Demand Curves

- In the paper on Adena Springs Ranch I need to approximate various marginal cost (firm level supply) and demand curves.
- However, like the production application above, I don't have a lot of data.
- A simple approximation is based on a Translog approximation with a "choke price"

$$MC(Q) = \alpha_0 + \exp[\alpha_1 + \alpha_2 \ln(Q)] \quad (4)$$

where α_0 is the "choke price".

- Given the choke price, I then solve for α_1 and α_2 using two conditions: (1) An observed price/quantity combination and (2) the slope or elasticity at the observed price/quantity combination.

- Taking $\alpha_0 = \tilde{\alpha}_0$, $Q = \tilde{Q}$, and $P = MC(Q) = \tilde{P}$, and assume that you know the elasticity of the supply function $\tilde{\epsilon}$ we can parameterize the supply function as

$$\begin{aligned}\tilde{\alpha}_0 + \exp(\alpha_1 + \alpha_2 \tilde{Q}) &= \tilde{P} \\ \frac{\partial MC(Q)}{\partial Q} &= \alpha_2 \exp(\alpha_1 + \alpha_2 \tilde{Q}) = \tilde{\epsilon} \frac{\tilde{P}}{\tilde{Q}}\end{aligned}\tag{5}$$