

Lecture XXXI: Input-Output Analysis, Leontief and Impact Multipliers

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1 Economic Tableau

Economic Tableau

- Economic theory seeks to explain the material aspects and operations of our society in terms of the intersection of supply and demand or wages and prices.
- An alternative way is to envision the economy as a set of transactions.
 - The horizontal rows are how the output for each sector is distributed among the others.
 - The columns show how the sector obtains its inputs from the other sectors.

Table: Quantity Tableau

	Sector 1: Agriculture	Sector 2: Manufacture	Sector 2: Household	Total Output	
Sector 1: Agriculture	25	20	55	100	Bu. of Wheat
Sector 2: Manufacture	14	6	30	50	Yards of cloth
Sector 3: Household	80	180	40	300	Man Years of Labor

- Although in principle the intersection flows as represented in an input-output table can be thought of as being measured physical units, in practice most input-output tables are constructed in value terms.
 - \$ 2/bu. of wheat
 - \$ 5/yard of cloth
 - \$ 1/man hour of labor

Table: Value Tableau

	Sector 1: Agriculture	Sector 2: Manufacture	Sector 2: Household	Total Output
Sector 1: Agriculture	50	40	110	200
Sector 2: Manufacture	70	30	150	150
Sector 3: Household	80	180	40	300
Total Input	220	250	300	

- Let the national economy be divided into $n + 1$ sectors with n industries that produce goods and sector $n + 1$ being the final demand.
- For purposes of mathematical manipulation, the physical output of sector i is usually represented by x_i while x_{ij} stands for the amount of the product of sector i absorbed by sector j .
- The quantity of the product of sector i delivered to the final demand sector $x_{i,n+1}$ is usually identified as y_i .
- The quantity of the output of sector i absorbed by sector j per unit of its total output j is depicted by the symbol a_{ij} and is called the input coefficient of the product of sector i into sector j .

$$a_{ij} = \frac{x_{ij}}{x_j} \quad (1)$$

Table: Value Tableau

	Sector 1: Agriculture	Sector 2: Manufacture	Sector 2: Household
Sector 1: Agriculture	0.25	0.40	0.133
Sector 2: Manufacture	0.14	0.12	0.100
Sector 3: Household	0.80	0.60	0.133

$$\begin{aligned}x_1 &= x_{11} + x_{12} + x_{13} + y_1 \\x_2 &= x_{21} + x_{22} + x_{23} + y_2 \\x_3 &= x_{31} + x_{32} + x_{33} + y_3\end{aligned}\tag{2}$$

$$\begin{aligned}x_1 &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + y_1 \\x_2 &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + y_2 \\x_3 &= a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + y_3\end{aligned}\tag{3}$$

$$\begin{aligned}(1 - a_{11})x_1 - a_{12}x_2 - a_{13}x_3 &= y_1 \\ a_{21}x_1 + (1 - a_{22})x_2 - a_{23}x_3 &= y_2 \\ a_{31}x_1 - a_{32}x_2 + (1 - a_{33})x_3 &= y_3\end{aligned}\tag{4}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\left[I - \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \left[I - \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \right]^{-1} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Table: Impact of Trade

	Sector 1: Agriculture	Sector 2: Manufacture	Sector 2: Household	Export (+)	Final Demand	Total Output
Sector 1: Agriculture	19.04	22.12	55	-20	35	76.1
Sector 2: Manufacture	10.66	6.64	30	+8	38	55.3
Sector 3: Households	60.93	199.07	40		40	300.