

Lecture XXVIII: An Overview of Computable General Equilibrium

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- 1 Partial Versus General Equilibrium
- 2 Empirical Model of General Equilibrium – Excess Demand
- 3 Existence of Walrasian Equilibrium

Partial Versus General Equilibrium

- Most of our policy analysis can be characterized as partial.
 - Consider the supply and demand for college degrees presented in Figure 1.
 - Implicitly, the Morrill Act of 1862 establishing the Land Grant University System subsidized college degrees by providing funds (from the sale of land – hence "Land Grant") for the establishment of universities such as the University of Florida.
 - Our partial analysis then typically proceeds by examining the effect of the subsidy. In this case the overall quantity of degrees increases from q_1^* to $\tilde{q}_1$. The difference in price is the amount of the subsidy $\tilde{p}_1^P - \tilde{p}_1^C$.

- The supply of college degrees ($S_1(p_1, w_1, w_2)$) is a function of price of college degrees and the price of inputs used to produce college degrees (w_1 – wage rate for professors and w_2 – cost of technology).
- The demand for college degrees ($D_1(p_1, p_2, z_1, z_2, z_3)$) is a function of the price of college degrees along with the price of other education opportunities (p_2), the wage rate for college educated individuals (z_1), the wage rate for other individuals (z_2 – trade school educations), and other factors (z_3).

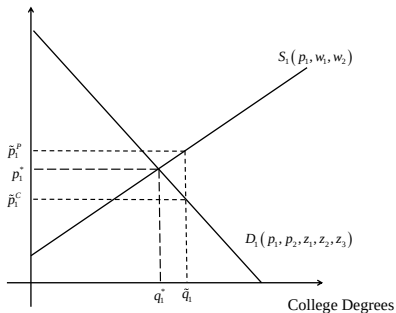


Figure: Supply and Demand of College Degrees

- In our partial analysis, we have not included the possible effect of the educational subsidy on other markets. For example, subsidizing college degrees increases the supply of educated individuals, but reduces the supply of tradesmen.
- As depicted in Figure 2, this implies changes in the return to education.
 - Subsidizing college degrees shifts the supply of educated labor to the right from $S_z^1(p_1^*, p_2, z_1, z_2, z_3)$ to $S_z^1(\tilde{p}_1^C, p_2, z_1, z_2, z_3)$. This results in a decline in the price of educated labor from z_1^* to $\tilde{z}_1$ and an increase in the quantity of educated labor consumed from x_1^* to $\tilde{x}_1$.
 - Subsidizing college degrees also causes the supply of tradesmen to shift to the left from $S_z^2(p_1^*, p_2, z_1, z_2, z_3)$ to $S_z^2(\tilde{p}_1^C, p_2, z_1, z_2, z_3)$ resulting in the consumption of less tradesman labor (x_2^* to $\tilde{x}_2$). As a result, the wage rate for tradesmen increases from z_2^* to $\tilde{z}_2$.

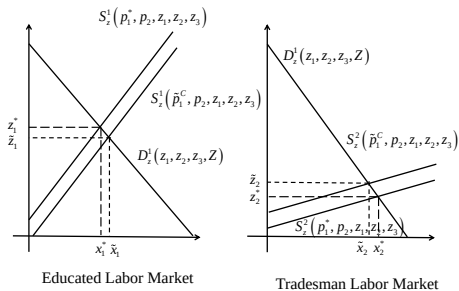


Figure: Labor Market

- The story is then one of attenuation. As the price in the educated labor market falls from z_1^* to $\tilde{z}_1$, the demand for education shifts to the left from $D_1(p_1, p_2, z_1^*, z_2^*, z_3)$ to $D_1(p_1, p_2, \tilde{z}_1, \tilde{z}_2, z_3)$ as depicted in Figure 3.
- Holding the level of subsidy constant, the quantity of college degrees drops from $\tilde{q}_1$ to $\hat{q}_1$ as the price paid by students falls from $\tilde{p}_1^C$ to $\hat{p}_1^C$.

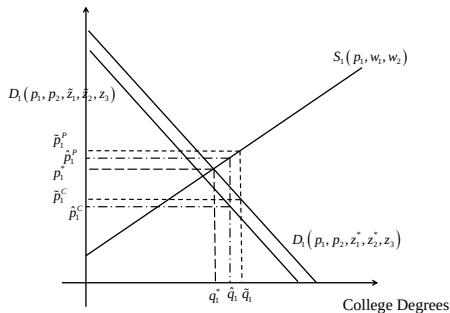


Figure: Supply and Demand for College Degrees – Response

- These interactions continue until the effect dies out – Walrassian Tatonnement.
- The interaction leads to other changes:
 - Possibly changes in aggregate income – Investment in human capital may be incorporated into a growth model.
 - The subsidy must be paid for – Implications of taxation.
 - Factor markets are also affected – What happens to the wages of college professors?

Empirical Model of General Equilibrium – Excess Demand

- Early work on general equilibrium analysis involves the concept of a Walrasian equilibrium.
 - The primary idea of the Walrasian equilibrium was the concept that some price vector could be found for any endowment that equated the supply and demand, or resulted in zero excess demand

$$\begin{aligned}\xi_i(p) &= D_i(p, W) - S_i(p, W) \\ p_i(\xi_i(p) - W_i) &= 0 \\ \xi_i(p) - W_i &\leq 0\end{aligned}\tag{1}$$

- $\xi_i(p)$ is the excess demand for good i , it is a function of the price vector p .
 - This demand is determined in part by the initial endowment of goods W .
- $p_i(\xi_i(p) - W_i)$ is the complementary slackness conditions.

- This condition implies that either the price of the i^{th} good is zero, or its excess demand is zero.
- Under the traditional formulation, Walrasian equilibria were hypothesized by tatonnement.
 - Following the preceding discussion, we could iterate through the equilibrium until all the excess demand curves met the Walrasian conditions.
 - Basically, economist simply assumed that the process would yield a viable solution.
 - Define a viable solution as one where a unique set of positive prices simultaneously solved for the Walrasian equilibrium.

- Wald, A. 1951. On Some Systems of Equations of Mathematical Economics. *Econometrica* 91, 368-403.
 - As a rule, economists have contented themselves with equating the number of equations and unknowns and have assumed, without further investigation, that the system of equations had a meaningful solution from an economic viewpoint, and that this solution was unique. But the equality of the number of equations and unknowns does not prove that a solution exists, much less the uniqueness of a solution (Wald pp 369-70).
- Arrow, K.J. and G. Debreu. 1954. Existence of an Equilibrium for a Competitive Economy. *Econometrica* 22, 265-290 – Offer a definitive proof of the existence of a solution to the general equilibrium problem based on what are referred to as Fixed Point Theorems.

- However, for applied economist knowing that an equilibrium exists is insufficient.
 - The real question is "what kinds of questions can I address with the procedure?" and "how do I find the equilibrium?"
 - In general there are two primary techniques for finding the equilibrium solution.
 - The first is attributed to Scarf, H. 1967. The Approximation of Fixed Points of a Continuous Mapping. *Journal of Applied Mathematics* 15, 1328-43.
 - This procedure involves decomposing the space into simplexes or triangularizations then searching the triangularizations by eliminating points.
 - The second set of methods are basically gradient search algorithms (Newton Raphson).
 - The simplex methodologies are sure in that they will always find the fixed point. However, they are also difficult to formulate. The Newton methods are far more straightforward to formulate, but may fail to find a solution if the general equilibrium is poorly structured.

Existence of Walrasian Equilibrium

- Following Varian, H. R. 1984. *Microeconomic Analysis* Second Edition, New York: W.W. Norton. with the notation from Shoven, J. B. and J. Whalley. 1992. *Applying General Equilibrium*, New York: Cambridge University Press. We begin by examining the allocation of outputs among n competing consumers.
- In this case, we assume that consumers start with an endowment of m goods.
 - The general equilibrium then involves trading the preexisting goods among themselves until an appropriate price vector is determined.
 - In this case, we eliminate the production activities from our earlier discussion to yield

$$\begin{aligned}\xi_i(p) &= D_i(p, W) - W_i \\ p_i \xi_i(p) &= 0 \\ \xi_i(p) &\leq 0\end{aligned}\tag{2}$$

- To graphically motivate this discussion, consider the allocation of two goods.
 - In this case the initial allocation of the goods produces the dimensions of the traditional Edgeworth box where

$$\begin{aligned}X_1 &= W_1^1 + W_1^2 \\X_2 &= W_2^1 + W_2^2\end{aligned}\tag{3}$$

given that W_i^j is the amount of good i held by agent j .

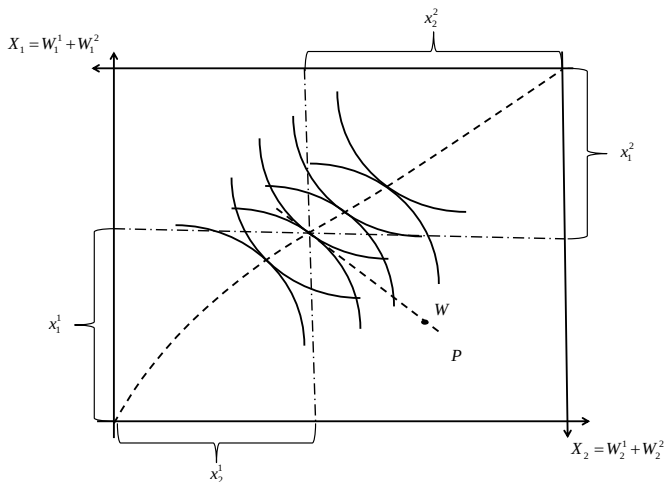


Figure: Edgeworth Box

- The existence of a unique general equilibrium solution is equivalent to the statement that there exists one set of prices that determine a line from the endowment point W to the locus of optimal tangencies.
- As a first step, we need to normalize the prices.
 - Specifically, given that demand functions are homogeneous of degree zero in prices (there is no money illusion) we must determine a numeraire, a price equal to one, or impose the restriction that the prices sum to some constant.
 - Varian takes the second approach

$$S^{k-1} = \left\{ p \text{ in } R_+^k : \sum_{i=1}^k p_i = 1 \right\} \quad (4)$$

- Restating the goal, we want to show that there exists a p defined above such that $p\xi(p) = 0$ for all goods with $p \geq 0$ and $\xi(p) \leq 0$.
 - From a mathematical point of view, not that p can be written as a function of excess demand and excess demand is a function of p

$$\begin{pmatrix} p & p(\xi) \\ \xi & \xi(p) \end{pmatrix} \text{ s.t. } p \rightarrow \xi(p) \text{ and } \xi \rightarrow p(\xi) \quad (5)$$

- Fixed point theorems show that under certain conditions p can be chosen such that $p \rightarrow \xi(p) \rightarrow p$, or that the mapping is "fixed".

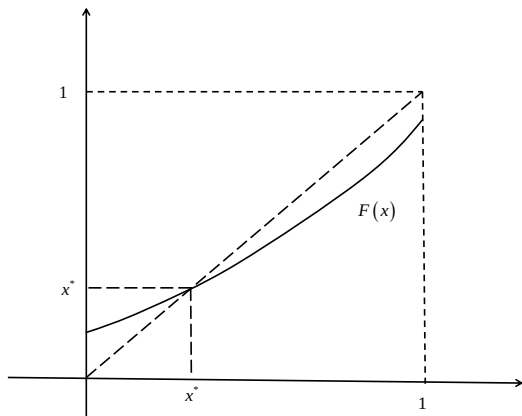


Figure: Fixed Point