

Lecture XXV: Midterm Examination and Gauss-Seidel

Charles B. Moss¹

¹University of Florida

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1 Midterm

2 Gauss Seidel

Simple Sector Level Model – Question 1

1. The gulf oil spill highlighted the possible effect of intermediate products necessary for producing consumption goods (oil) on environmental quality. Consider the following problem

$$\begin{aligned}
 & \max_{y_2, y_3} y_2^{0.3} y_3^{0.5} \\
 \text{s.t. } & y_1 = (0.39685x_{11}^{0.3333} + 0.20083x_{21}^{0.3333} + 0.11696x_{31}^{0.3333})^{-3} \\
 & y_2 = (0.13442y_{21}^{0.6667} + 0.13442x_{12}^{0.6667} + 0.068400x_{22}^{0.6667} + 0.02154x_{32}^{0.6667})^{-1.5} \\
 & y_3 = (0.13337y_{31}^{0.25} + 0.3181x_{13}^{0.25} + 0.13375x_{23}^{0.25} + 0.13375x_{33}^{0.25})^{-4.0} \\
 & y_{21} + y_{31} \leq y_1 \\
 & x_{11} + x_{12} + x_{13} \leq 20 \\
 & x_{21} + x_{22} + x_{23} \leq 15 \\
 & x_{31} + x_{32} + x_{33} \leq 20.
 \end{aligned} \tag{1}$$

Solve the economic problem in Equation 1 using your choice of solver (30 points).

- Next, we modify this specification to account for an externality associated with the production of y_1 . Specifically, solve the same problem with the objective function

$$\max_{y_1, y_2} y_2^{0.3} y_3^{0.5} y_1^{-0.2}. \quad (2)$$

How does the use of resources change in this solution? (25 points)

Simple Sector Model – Question 2

2. Maximize the profit of production based on the endowments specified as

$$\max_{y_1, y_2, x_1, x_2, x_3} 10y_1 + 15y_2$$

$$y_1 = f_1(x_1, x_2, x_3) = \exp[0.05 + 0.30 \ln(x_1) + 0.25 \ln(x_2) + 0.15 \ln(x_3) - 0.010 \ln(x_1) \ln(x_1) + 0.001 \ln(x_1) \ln(x_2) - 0.002 \ln(x_1) \ln(x_2) - 0.025 \ln(x_2) \ln(x_2) + 0.005 \ln(x_2) \ln(x_3) - 0.009 \ln(x_3) \ln(x_3)]$$

$$y_2 = f_2[y_1, x_1, x_2, x_3] = \exp[0.03 + 0.40 \ln(y_1) + 0.20 \ln(x_1) + 0.10 \ln(x_2) + 0.05 \ln(x_3) - 0.001 \ln(y_1) \ln(y_1) + 0.0005 \ln(y_1) \ln(x_1) - 0.0002 \ln(y_1) \ln(x_2) + 0.0009 \ln(y_1) \ln(x_3) - 0.0009 \ln(x_1) \ln(x_1) + 0.0006 \ln(x_1) \ln(x_2) + 0.0010 \ln(x_1) \ln(x_3) - 0.0012 \ln(x_2) \ln(x_2) + 0.0003 \ln(x_2) \ln(x_3) - 0.0001 \ln(x_3) \ln(x_3)]$$

$$x_{11} + x_{12} \leq 20, \quad x_{21} + x_{22} \leq 10, \quad x_{31} + x_{32} \leq 8$$

(3)

using Minos 5.5 (25 points). Demonstrate that the optimal point meets the first and second-order necessary conditions (20 points).

Gauss Seidel

- Suppose that we wanted to solve for the x such that $G(x) = b$.
- For example, take a system of CES production functions

$$\begin{aligned}g_1(x) &= (0.5398x_1^{0.4286} + 0.6255x_2^{0.4286} + 0.4210x_3^{0.4286})^{2.333} = 5 \\g_2(x) &= (0.5428x_1^{0.3333} + 0.2154x_2^{0.3333} + 0.6300x_3^{0.3333})^{3.000} = 8 \\g_3(x) &= (0.2759x_1^{0.2000} + 0.4804x_2^{0.2000} + 0.4805x_3^{0.2000})^{5.000} = 4.5\end{aligned}\tag{4}$$

- To solve this expression, we take the first-order expansion

$$\begin{aligned}G(x) &= G(x_0) + \nabla_x G(x_0)(x - x_0) = b \\ \Rightarrow x_1 &= x_0 + [\nabla_x G(x_0)]^{-1}(G(x_0) - b)\end{aligned}\tag{5}$$