

Lecture XXIV: An Introduction to GAMS

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1 Simple Sector Level Model

- Sector Level Profit

Simple Sector Level Model

- Let's start with two production functions.
- We assume that x_1 - capital, x_2 - labor, and x_3 - energy are fixed to the overall economy (initial endowments).
- We use those inputs to manufacture y_1 - materials and y_2 - manufactured goods.
- In addition, we assume that materials are required inputs in the production of manufactured goods

$$\begin{aligned} f_1(x_1, x_2, x_3) = & \exp [0.05 + 0.30 \ln(x_1) + 0.25 \ln(x_2) + 0.15 \ln(x_3) \\ & - 0.010 \ln(x_1) \ln(x_1) + 0.001 \ln(x_1) \ln(x_2) - 0.002 \ln(x_1) \ln(x_2) \\ & - 0.025 \ln(x_2) \ln(x_2) + 0.005 \ln(x_2) \ln(x_3) - 0.009 \ln(x_3) \ln(x_3)] \end{aligned}$$

$$\begin{aligned} f_2(y_1, x_1, x_2, x_3) = & \exp [0.03 + 0.40 \ln(y_1) + 0.20 \ln(x_1) + 0.10 \ln(x_2) + 0.05 \ln(x_3) \\ & - 0.001 \ln(y_1) \ln(y_1) + 0.0005 \ln(y_1) \ln(x_1) - 0.0002 \ln(y_1) \ln(x_2) + 0.0009 \ln(y_1) \ln(x_3) \\ & - 0.0009 \ln(x_1) \ln(x_1) + 0.0006 \ln(x_1) \ln(x_2) + 0.0010 \ln(x_1) \ln(x_3) - 0.0012 \ln(x_2) \ln(x_2) \\ & + 0.0003 \ln(x_2) \ln(x_3) - 0.0001 \ln(x_3) \ln(x_3)] \end{aligned} \tag{1}$$

Sector Level Profit

- As an initial concept, assume that the sector maximized profit (eventually we will let the input prices be endogenously determined).
- First, consider the problems separately.

$$\max 10f_1(x_1, x_2, x_3) - 0.50x_1 - 0.95x_2 - 1.5x_3 \Rightarrow \begin{bmatrix} x_1 = 20.197 \\ x_2 = 6.991 \\ x_3 = 3.835 \end{bmatrix} \quad (2)$$

$$\max 15f_2(y_1, x_1, x_2, x_3) - 10.0y_1 - 0.50x_1 - 0.95x_2 - 1.5x_3 \Rightarrow \begin{bmatrix} y_1 = 1.278 \\ x_1 = 12.528 \\ x_2 = 3.311 \\ x_3 = 1.129 \end{bmatrix} \quad (3)$$

- Next, we combine the problems

$$\left. \begin{array}{l} \max 15f_2(y_{11}, x_{11}, x_{21}, x_{31}) + 10f_1(x_{21}, x_{22}, x_{32}) \\ -0.50(x_{11} + x_{21}) - 0.95(x_{12} + x_{22}) - 1.5(x_{31} + x_{32}) \end{array} \right\} \quad (4)$$
$$\Rightarrow \left[\begin{array}{ll} y_{11} = 1.278 & \\ x_{11} = 12.528 & x_{12} = 20.197 \\ x_{21} = 3.311 & x_{22} = 6.991 \\ x_{31} = 1.129 & x_{32} = 3.835 \end{array} \right]$$