

Lecture XXIII: Advanced Topics in Nonlinear Optimization—Interior Point Algorithms

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1 Interior Point Algorithms

Start by considering a simple linear programming problem

$$\begin{aligned} \max_{x_1, x_2} & 10x_1 + 8x_2 \\ \text{s.t.} & x_1 + 3x_2 \leq 7 \\ & x_1 + x_2 \leq 5 \\ & 4x_1 + 3x_2 \leq 12 \end{aligned} \tag{1}$$

This problem can be easily solved using the simplex algorithm developed by Danzig (basically a system of elementary row operations).

Simples - Iteration 1

	x_1	x_2	s_1	s_2	s_3	rhs	γ_i
obj	-10.00	-8.00	0.00	0.00	0.00	0.00	
c_1	1.00	3.00	1.00	0.00	0.00	7.00	7.00
c_2	1.00	1.00	0.00	1.00	0.00	5.00	5.00
c_3	4.00	3.00	0.00	0.00	1.00	12.00	3.00

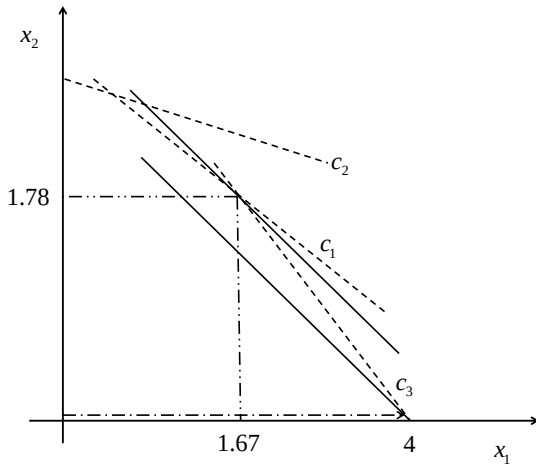
Simplex - Iteration 2

	x_1	x_2	s_1	s_2	s_3	rhs	γ_i
obj	0.00	-0.50	0.00	0.00	0.00	2.50	
c_1	0.00	2.25	1.00	0.00	-0.25	4.00	1.78
c_2	0.00	0.25	0.00	1.00	-0.25	2.00	8.00
x_1	1.00	0.75	0.00	0.00	0.25	3.00	4.00

Simplex - Iteration 3

	x_1	x_2	s_1	s_2	s_3	rhs	γ_i
obj	0.00	0.00	0.22	0.00	2.44	30.89	
x_2	0.00	1.00	0.44	0.00	-0.11	1.78	
c_2	0.00	0.00	-0.11	1.00	-0.22	1.56	
x_1	1.00	0.00	-0.33	0.00	0.33	1.67	

Simplex Solution



Consider rewriting the optimization problem in Equation 1 as a barrier function

$$\max_{x_1, x_2} 10x_1 + 8x_2 - \frac{r_1}{7 - x_1 - 3x_2} - \frac{r_2}{5 - x_1 - x_2} - \frac{r_3}{12 - 4x_1 - 3x_2} \quad (2)$$

Iterations

- Starting at $x = (1, 1)$ and $r = (100, 100, 100)$, the Newton-Raphson procedure yields the results in Table 10.

First Penalty Function

Iter	x_1	x_2	$\partial f / \partial x_1$	$\partial f / \partial x_2$	Norm
1	0.8897	0.5133	-10.9150	-20.3902	23.1279
2	1.2575	-0.4286	-3.6401	-8.2217	8.9915
3	2.5699	-2.1976	-1.2805	-3.4795	3.7077
4	4.9886	-5.2073	-0.7974	-1.7397	1.9137
5	8.4140	-9.6014	-0.5739	-0.8833	1.0534
6	11.8804	-14.1557	-0.2521	-0.3207	0.4079
7	13.8271	-16.7373	-0.0437	-0.0519	0.0679
8	14.2103	-17.2469	-0.0013	-0.0015	0.0020

Iterations

- Starting at $x = (1, 1)$ and $r = (100, 100, 100)$, the Newton-Raphson procedure yields the results in Table 10.
- Next, we reduce the barrier parameters to $r = (50, 50, 50)$ and resolving the problem yields the solution in Table 12.

Second Penalty Function

Iter	x_1	x_2	$\partial f / \partial x_1$	$\partial f / \partial x_2$	Norm
1	6.4082	-5.6824	-10.0501	-8.0833	12.8974
2	8.2216	-8.3920	-2.8278	-2.2845	3.6353
3	9.3852	-10.1015	-0.4078	-0.3327	0.5263
4	9.6746	-10.5176	-0.0117	-0.0098	0.0153
5	9.6876	-10.5357	0.0000	0.0000	0.0000

- Starting at $x = (1, 1)$ and $r = (100, 100, 100)$, the Newton-Raphson procedure yields the results in Table 10.
- Next, we reduce the barrier parameters to $r = (50, 50, 50)$ and resolving the problem yields the solution in Table 12.
- When $r = (1.5625, 1.5625, 1.5624)$ we have the results in Table 14.
- In fact, when $r = (0.04882, 0.04882, 0.04882)$ the solution is the same as for the simplex.
- This solution mechanism is presented graphically in Figure 15.

Third Penalty Function

Iter	x_1	x_2	$\partial f / \partial x_1$	$\partial f / \partial x_2$	Norm
1	2.0347	1.1001	-10.8786	-9.0393	14.1440
2	2.1757	0.8629	-3.1743	-2.6857	4.1580
3	2.3048	0.6623	-0.5041	-0.4519	0.6770
4	2.3658	0.5748	-0.0191	-0.0216	0.0288
5	2.3738	0.5639	-0.0001	-0.0001	0.0001

Interior Point Algorithm

