

Lecture XX: Nonlinear Constraints – Barrier Function and Projected Augmented Lagrangian

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1 Genetic Algorithms

Genetic Algorithm

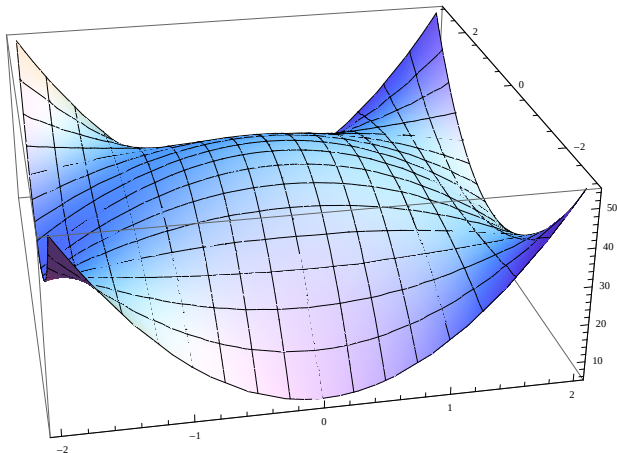
- As a starting point, consider the slightly complex function

$$g(x_1, x_2) = (2x_1^2 - 4)(x_2^2 - 3) + 30 \quad (1)$$

depicted in Figure 4.

- Intuitively, this function is not going to work well with the gradient algorithms we have been working with. One alternative comes from the mathematics of evolution.
- Consider the series of “genetic solutions” presented in Table 5.

Sombrero Function



Genetic Solutions

Individual	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	1	0	0	0	1	1	0	1	0	0	1	0	1	0
2	1	1	1	0	1	1	0	1	0	0	1	1	0	1	0	0
3	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0
4	0	0	0	1	0	0	0	1	1	1	0	0	0	1	0	1
5	1	0	0	0	1	1	0	0	1	0	0	1	1	0	0	1
6	0	0	1	0	0	1	1	1	0	1	1	0	1	1	1	0
7	1	0	1	1	1	1	0	1	0	0	0	0	1	0	1	1
8	0	1	1	1	1	0	0	0	1	0	1	0	1	0	1	1
9	1	0	0	1	0	1	0	1	0	1	1	0	0	0	0	1
10	0	1	1	0	0	1	1	1	1	0	0	1	1	1	1	1

- Let us assume that the genetic code in columns 1-8 are for variable x_1 and 9-16 are for variable x_2 . We can recover an integer from this sequence based on the binary formulation of the integer

$$z_{kj} = \sum_{i=1}^8 t_{ij} 2^{8-i} \quad k = 1, 2 \quad j = 1, 2, \dots, 36 \quad (2)$$

- Thus, Table 5 maps into the integers presented in the first two columns Table 7.
- We can map these integers into the range of our function

$$\begin{aligned} x_{1j} &= -2 + \frac{4}{255} z_{1j} \\ x_{2j} &= -3 + \frac{6}{255} z_{2j} \end{aligned} \quad (3)$$

Integer Numbers from the Binary Genetic Codes

Individual	z_1	z_2	x_1	x_2	$g(x_1, x_2)$
1	163	74	0.556863	-1.25882	34.7837
2	237	52	1.71765	-1.77647	30.2962
3	254	192	1.98431	1.51765	27.3001
4	17	197	-1.73333	1.63529	29.3455
5	140	153	0.196078	0.60000	40.357
6	39	110	-1.38824	-0.411765	30.4121
7	189	11	0.964706	-2.74118	20.3459
8	120	171	-0.117647	1.02353	37.7555
9	149	97	0.337255	-0.717647	39.3746
10	103	159	-0.384314	0.741176	39.0787

- Columns 3 and 4 give the values of real variables x_1 and x_2 . Finally, column 5 gives the value of the function for these real variables.

- All of this was simply setup. Next, we need to consider “mating” the points.
- To to this, I want to pair up points based on the relative goodness of each point. We measure goodness by the relative size of the objective function

$$p_i = \frac{g(x_{1i}, x_{2i})}{\sum_{j=1}^{36} g(x_{1j}, x_{2j})} \quad (4)$$

Probability of Breeding

Ind	$g(x_1, x_2)$	p_i	Ind	$g(x_1, x_2)$	p_i	Ind	$g(x_1, x_2)$	p_i
1	34.7837	0.0323143	13	31.3439	0.0291187	25	41.4546	0.0385116
2	30.2962	0.0281454	14	29.3294	0.0272472	26	20.0767	0.0186514
3	27.3001	0.0253620	15	29.1195	0.0270522	27	39.3570	0.0365629
4	29.3455	0.0272622	16	38.2210	0.0355076	28	30.7401	0.0285578
5	40.3570	0.0374920	17	29.6705	0.0275641	29	41.0975	0.0381799
6	30.4121	0.0282531	18	39.2089	0.0364254	30	19.4868	0.0181034
7	20.3459	0.0189015	19	26.7884	0.0248867	31	27.8551	0.0258776
8	37.7555	0.0350752	20	18.6384	0.0173152	32	30.7455	0.0285628
9	39.3746	0.0365794	21	21.9806	0.0204202	33	24.0960	0.0223853
10	39.0787	0.0363044	22	28.2260	0.0262221	34	24.0245	0.0223189
11	22.5457	0.0209451	23	32.1844	0.0298996	35	34.1496	0.0317253
12	23.9608	0.0222597	24	12.8438	0.0119320	36	30.2235	0.0280779

Breeding Pairs

Pair	I_1	I_2	Pair	I_1	I_2
1	5	7	26	16	27
2	22	8	27	13	6
3	26	10	28	17	36
4	32	22	29	5	5
5	36	27	30	24	8
6	4	10	31	14	14
7	28	2	32	7	29
8	11	25	33	15	18
9	25	15	34	4	28
10	23	34	35	16	18
11	4	15	36	28	11
12	25	5	37	28	5
13	14	32	38	22	3
14	26	24	39	3	20
15	10	16	40	23	32
16	35	10	41	6	31
17	33	19	42	13	29
18	18	22	43	18	8
19	4	4	44	9	36
20	24	28	45	2	16
21	3	31	50	1	32

- Consider the first breeding pair presented in Table 13.
- We generate a cross-over point which is a random integer between 1 and 16 - Say 10.
- The “cross-over” is presented in Table 14.
- These values are mapped back into numbers in Table 15.

Breeding Pair

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
5	1	0	0	0	1	1	0	0	1	0	0	1	1	0	0	1
7	1	0	1	1	1	1	0	1	0	0	0	0	1	0	1	1

New Individuals

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	0	0	1	1	0	0	1	0	0	0	1	0	1	1
2	1	0	1	1	1	1	0	1	0	0	0	1	1	0	0	1

Numeric Results of Pairing

l	z_1	z_2	x_1	x_2	$g(x_1, x_2)$
1	169	134	0.650980	0.152941	39.3836
2	147	188	0.305882	1.423530	33.7121
3	233	1	1.654900	-2.976470	38.6566
4	183	96	0.870588	-0.741176	36.0878
5	105	219	-0.352941	2.152940	23.8668
6	63	153	-1.011760	0.600000	35.1550
7	228	22	1.576470	-2.482350	33.0689
8	183	204	0.870588	1.800000	29.4038
9	67	186	-0.949020	1.376470	32.4303
10	230	183	1.607840	1.305880	28.4848
11	228	246	1.576470	2.788240	34.6335
12	65	186	-0.980392	1.376470	32.2965
13	42	199	-1.341180	1.682350	30.0683
14	86	2	-0.650980	-2.952940	11.9684
15	188	208	0.949020	1.894120	28.7079
16	76	153	-0.807843	0.600000	37.1142
17	230	33	1.607840	-2.223530	32.2752
18	216	27	1.388240	-2.364710	29.6226
19	231	135	1.623530	0.176471	26.2245
20	140	217	0.196078	2.105880	24.3714
21	100	204	-0.431373	1.800000	29.1293
22	26	199	-1.592160	1.682350	29.8184
23	174	25	0.729412	-2.411760	21.7307
24	169	212	0.650980	1.988240	26.9955
25	26	75	-1.592160	-1.235290	28.4229

Results

I	New	Old	I	New	Old
1	28.6436	12.8438	19	32.6820	30.2235
2	28.7079	18.6384	20	32.7567	30.2962
3	29.0706	19.4868	21	32.8024	30.4121
4	29.1293	20.0767	22	33.0689	30.7401
5	29.4038	20.3459	23	33.1346	30.7455
6	29.6226	21.9806	24	33.7121	31.3439
7	29.7668	22.5457	25	33.7638	32.1844
8	29.7822	23.9608	26	34.6335	34.1496
9	29.8184	24.0245	27	35.1550	34.7837
10	29.8612	24.0960	28	36.0878	37.7555
11	29.9769	26.7884	29	37.1142	38.2210
12	30.0683	27.3001	30	37.1475	39.0787
13	30.2146	27.8551	31	38.4598	39.2089
14	30.7263	28.2260	32	38.6566	39.3570
15	31.3924	29.1195	33	39.2767	39.3746
16	32.2752	29.3294	34	39.3836	40.3570
17	32.2965	29.3455	35	39.3836	41.0975
18	32.4303	29.6705	36	41.9725	41.4546