

Lecture VII: Numeric Integrals - Historical Cost versus Market Value Accounting

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September 6, 2012

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Predictive Use of Financial Statements in Making a Loan

- Assume that a farmer applies for a loan with a commercial bank. Typically, the banker and farmer attempt to estimate his income and equity position.
- The banker and farmer could construct these financial statements using either accrual accounting system relying on historical cost (psuedo Generally Accepted Accounting Principles [GAAP] compliant) or market valued principles.
 - It should be noted that the information would probably not be available for the construction of fully GAAP compliant statements. The matching of cash expenditures on crops with each crop's disposition would be a daunting task for the smallest of farms.
 - Even if those statements could be constructed, it is highly unlikely that the statements could ever reach the standards of being auditable. Auditing requires the establishment of procedures for recognizing accounting events.

- The relevant question is then which accounting statements are more useful?
 - If the accrual accounting statements based on historical cost are more useful, then the question would be whether the value added by GAAP compliant statements exceed the cost of maintaining such systems at the farm level.
 - Alternatively, it is possible that the market valued financial statements are more useful.

For our purpose here, assume that the banker is interested in whether the farmer would be able to repay the principal and interest of an operating loan (P) at the end of the period from his income from agricultural operations (R) given his consumption requirements (C). Mathematically, the farmer would be able to pay the loan if

$$R - P - C \geq 0 \Rightarrow R \geq P + C. \quad (1)$$

Recognizing that income is a random variable, the lender will make the loan if

$$P [R \geq P + C] \geq \delta \quad (2)$$

where $P []$ denotes the probability and δ is some level of probability set by the bank policy (say $\delta = 0.95$).

Defining $f [R_i]$ as the probability distribution function (i.e., the probability that $R = R_i$), the probability that the farmer will be able to make the loan payment can be defined as

$$P [R \geq P + C] = \sum_{i \in R_i \geq P+C} f [R_i]. \quad (3)$$

Intuitively, this probability is a function of other factors.

For example, the probability of obtaining a certain income this year (R_t) may be a function of income last year (R_{t-1}), the level of assets last year (or better yet at the end of the last year/beginning of this year V_{t-1}), other debts owed by the farmer at the beginning of the year (D_{t-1}), etc. Thus, we can rewrite the probability distribution function as

$$f[R_i] \Rightarrow f[R_{it} | R_{t-1}, V_{t-1}, D_{t-1}, a_t] \quad (4)$$

where a_t is a host of other factors not included in the financial statements.

The two specifications may yield different expectations of the probability of repayment

$$\begin{aligned}
 P_M \left[R_t \geq C + P \mid R_{t-1}, V_{t-1}, D_{t-1}^*, a_t \right] &= \\
 \sum_{i \in R_{it} \geq C+P} f \left[R_{it} \mid R_{t-1}, V_{t-1}, D_{t-1}^*, a_t \right] &= \delta_M \\
 \\
 P_C \left[\tilde{R}_t \geq C + P \mid \tilde{R}_{t-1}, \tilde{V}_{t-1}, D_{t-1}^*, a_t \right] &= \\
 \sum_{i \in \tilde{R}_{it} \geq C+P} g \left[\tilde{R}_{it} \mid \tilde{R}_{t-1}, \tilde{V}_{t-1}, D_{t-1}^*, a_t \right] &= \delta_C
 \end{aligned} \tag{5}$$

The point here is that there is a true probability of repayment that we estimate with two different procedures. The question is really which estimation is more reliable or more useful?

Like the standard statistical t-test, we can estimate the probability that the farm will be able to make its loan payments under each estimate as

$$\hat{\delta}_C = \Phi \left(\frac{\bar{R}_C - [C + P]}{\sqrt{\sigma_C^2}} \right)$$
$$\hat{\delta}_M = \Phi \left(\frac{\bar{R}_M - [C + P]}{\sqrt{\sigma_M^2}} \right)$$
(6)

where $\Phi []$ is the cumulative distribution for the standard normal distribution.

Simulated Annealing

We are interested in working with three equations. First, we hypothesize that returns follow a random walk process

$$r_{it} = \alpha_i + \beta_i \xi_t + \xi_{it} \quad (7)$$

where r_{it} is the rate of return on agricultural assets for firm i at time t , α_i is firm specific mean for firm i , β_i is a single index relationship between the randomness in the return for firm i and the randomness of the overall market, ξ_t is the common randomness in the overall market, and ξ_{it} is the randomness specific to firm i .

The overall market return is then formulated as

$$r_t = \alpha + \xi_t \ni \alpha = \frac{1}{N} \sum_{i=1}^N \alpha_{it}. \quad (8)$$

Given the overall market return, we assume that asset values follow a geometric Brownian motion process

$$\frac{dV}{V} = r_t - k_t + \nu_t \quad (9)$$

where V is the value of farmland in dollars per acre, r_t is the observed average return, k_t is a cost of capital, and ν_t is a random normal variate.

The final equation is then the consumption equation from Ramirez, Moss, and Boggess (1997)

$$C_t = E_t \left[\frac{\rho - k_t b}{(1 - b)} - \frac{b(k_t - \mu_{at})^2}{2(1 - b)\sigma_{At}^2} \right] \quad (10)$$

where ρ is the decision maker's rate of time preference, b is the decision maker's risk aversion coefficient, μ_{at} is the decision maker's expected return on assets (we will set this equal to α_i), and σ_{At}^2 is the decision maker's variance on asset returns (we will set this equal to $\beta_i^2 \sigma^2 + \sigma_i^2$ or the sum of the variance from the common factor and the individual's idiosyncratic variance).

These data can be defined for Florida using the ARMS data for 2003 through 2010 as presented in Table 15. By assumption, we will allocate one half of the variance of returns to the common variation and one half to idiosyncratic risk. The risk aversion coefficient is solved by inverting the optimal debt position. Ramirez, Moss, and Boggess derive the optimal debt-to-asset ratio to be

$$\delta_t^* = 1 - \frac{(1 - b) \sigma_{At}^2}{\mu_{At} - k_t} \quad (11)$$

solving this expression for b yields

$$b = 1 - \frac{(1 - \delta) (\mu_{At} - k_t)}{\sigma_{At}^2} \quad (12)$$

where $k_t = 0.02$.

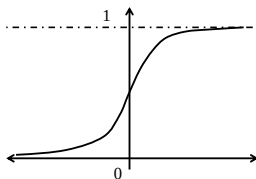
Florida Data

Year	Return	Asset Values	Rate of Return	Debt to Asset	Risk Aversion (<i>b</i>)	Land Values	Net Change
2002						2,720	
2003	27,712	668,179	0.04147	0.09042	-85.356	2,900	0.04260
2004	26,639	1,102,502	0.02416	0.03065	-91.031	3,100	0.06253
2005	30,339	1,666,244	0.01821	0.02151	-91.898	4,790	0.43692
2006	40,356	1,707,458	0.02364	0.02592	-91.480	5,320	0.10131
2007	38,559	1,333,905	0.02891	0.05052	-89.145	5,500	0.02437
2008	47,314	1,497,129	0.03160	0.03439	-90.676	5,640	0.01353
2009	63,159	1,184,860	0.05331	0.04706	-89.473	5,150	-0.12419
2010	53,420	1,348,258	0.03962	0.04301	-89.858	4,800	-0.09000
Mean	40,937	1,313,567	0.03261	0.04293	-89.865	4650.000	0.05838
Std Dev	13,027	336,470	0.01153	0.02172	2.062	1062.921	0.17068

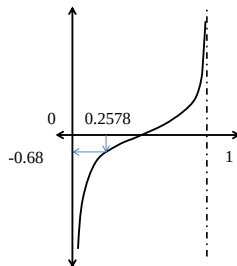
To construct the farm level results, we start by drawing different farms. To do this we draw α_i , β_i and b_i (for our purpose here we will set $\rho = 0.03$ and $k_t = 0.02$). The data presented in Table 15 provide guidance for the distributions of α_i and b_i . Since we want idiosyncratic risk to cancel out, we want the average β_i to be 1. As a starting point, we assume the variance of β_i to be 0.15. Under our baseline results, $C_t = 0.0266$ which implies a consumption of \$ 33,475 at the average equity position. How do we draw these values. The basic idea behind Simulated Annealing is drawing random numbers. In this case, we want to draw 250 normals (not necessarily correlated).

Most computer programs have uniform random number generators (Excel and Fortran). They do not have normal random number generators. There are several approaches to generating normal random numbers, we will use the inverse function formulation. We will draw a uniform random variable in Fortran and then determine the cumulative value of the normal that would yield that probability. The procedure is graphically depicted in Figure 3

Inverse Function Formulation



Cumulative Normal
Density Function



Inverse Cumulative
Normal Density
Function

Table 1 presents a synopsis of the individual firms. Using the firms, we can then draw 25 years of data for each of the variables. As a starting point, consider the results for income and farmland values. As a first step, we draw the common component of returns

$$\xi_t = (0.25 \times 0.01153) \epsilon_{1t} \quad (13)$$

where ϵ_{1t} is a standard normal distribution. Next, we draw each individual firm's return

$$r_{it} = \alpha_i + \beta_i \times \xi_t + (0.75 \times 0.01153) \epsilon_{2t} \quad (14)$$

where ϵ_{2t} is another standard normal variable.

Table: Random Firm Draws

Draw	α_i	β_i	b_i
1	0.29383489E-01	0.10039910E+01	-0.90079188E+02
2	0.32846847E-01	0.99552833E+00	-0.93317930E+02
3	0.33949277E-01	0.10026855E+01	-0.86748064E+02
4	0.31755884E-01	0.10002811E+01	-0.87889719E+02
5	0.27370927E-01	0.99070585E+00	-0.89711379E+02
6	0.33887501E-01	0.98602911E+00	-0.90476460E+02
7	0.28287182E-01	0.99879949E+00	-0.88861260E+02
8	0.33258019E-01	0.10085255E+01	-0.87761183E+02
9	0.32804774E-01	0.99233849E+00	-0.89048453E+02
⋮	⋮	⋮	⋮
250	0.30678922E-01	0.10022455E+01	-0.95202366E+02
Mean	0.0327	0.9996	-89.9562
Std. Dev.	0.0030	0.0054	1.9959

Next, we average the individual returns to the aggregate return

$$r_t = \frac{1}{250} \sum_{i=1}^2 50r_{it}. \quad (15)$$

This average return is used to generate the aggregate change in land values

$$\frac{\Delta V_i}{V_i} = (r_t - 0.02) + 0.007\nu_t \quad (16)$$

where ν_t is the standard normal. This change in land values then gives the actual land values

$$V_t = V_{t-1} \times \left(1 + \frac{\Delta V_t}{V_t}\right) \quad (17)$$