

Lecture III: Introduction to Computational Problems in Economics – Computable General Equilibrium

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August 27, 2012

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Under What Conditions Would Ethanol be in Society's Best Interest?

This study analyzes whether the introduction of ethanol provides benefits to the U.S. economy using a computable general equilibrium (CGE) approach based on data from the capital, labor, energy, and materials (KLEM) dataset of Dale Jorgenson (2010). This framework allows for the analysis of several of the consumer tradeoffs between food and fuel opportunities by directly modeling the interaction between each demand relationship.

The starting point of the analysis was the partial equilibrium analysis that Professor Schmitz and I did in "Ethanol: No Free Lunch." The earlier study examined the welfare cost of ethanol production under the 2002 Farm bill. As depicted in Figure 1 we defined the changes in consumer surplus, producer surplus and taxpayer spending. Estimates of each effect are presented in Table 1.

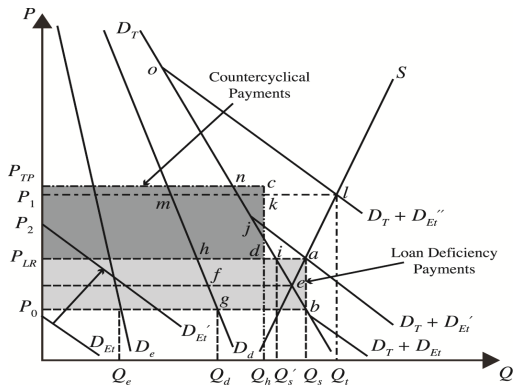


Figure: Effect of Increased Corn Demand from Ethanol on Corn Production with U.S. Agricultural Policies

Table: Welfare Impacts of a Shift in Corn Demand

Welfare Impact	Elasticity of Demand			
	0.4	0.5	0.6	0.7
Food/Alcohol/Industrial	-928.66	-816.96	-729.24	-658.52
Seed Use	9.79	8.55	7.59	6.82
Feed/Residual Use	-2,550.52	-2,258.79	-2,026.70	-1,837.73
Export	-874.14	-768.86	-686.22	-619.61
Producer Welfare Impact	975.80	884.45	808.20	743.89
Reduction in Direct Payments	3,765.57	3,300.61	2,297.20	2,645.34
Agg. Net Gain	-1,822.75	-1,882.31	1,928.84	-1,955.20
Net Gain to U.S.	-949.00	-1,113.00	-1,246.00	-1,346.00

Table: Added Effects of Ethanol Measured in the Fuel Market

Welfare Impact	Elasticity of Demand			
	0.4	0.5	0.6	0.7
Gasoline Price	2.969	2.969	2.969	2.969
Gasoline Quantity	139.726	139.630	139.733	139.733
Gain in Consumer Surplus	4.369	4.390	4.411	4.411
Loss to Gasoline & Oil Producers	-4.358	-4.378	-4.399	-4.399
Foreign Producers	-3.042	-3.057	-3.071	-3.071
Domestic Producers	-1.307	-1.314	-1.320	-1.320
Gain to Ethanol Producers	0.046	0.046	0.046	0.046
Net Welfare Gain	3.107	3.122	3.138	3.138

Table: Summary of Welfare Costs and Benefits: U.S. Ethanol Production

	Gain	Loss
Food/Alcohol/Industrial Use		-1,008
Feed/Residual Use		-3,094
Producer Impact	1,154	
Treasury Costs	4,084	
Ethanol Tax Credit		-2,761
Change in Surplus in Fuel Market	3,883	
Other	16	
Total	9,137	-6,863
Net Gain (Domestic)	+2,274	
Foreign Net Cost		-993
Net Gain (Global)	+1,281	

Moving from the partial equilibrium analysis, we constructed a computable general equilibrium model. Figure 2 depicts the tradeoff between energy, ethanol, and agricultural outputs. This figure depicts a "corner solution" where no ethanol is produced. Figure 4 depicts the change in equilibrium such that a subsidy (tax credit) is given to ethanol of sufficient amount to bring ethanol to the market.

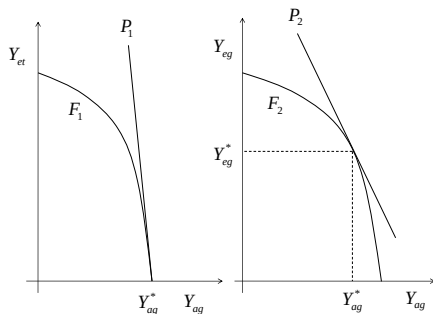


Figure: Simplified CGE Original Equilibrium

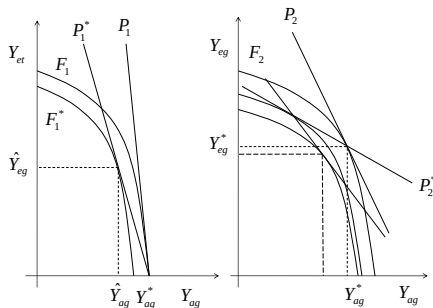


Figure: Subsidizing Ethanol Production

The original equilibrium is depicted by the conditions

$$\begin{aligned}\frac{\partial Y_{eg}}{\partial Y_{ag}} &= -\frac{p_{ag}}{p_{eg}} = P_1 \\ \frac{\partial Y_{eg}}{\partial Y_{et}} &= -\frac{p_{et}}{p_{eg}} = P_2\end{aligned}\quad (1)$$

after the ethanol subsidy the general equilibrium conditions become

$$\begin{aligned}\frac{\partial Y_{eg}}{\partial Y_{ag}} &= -\frac{p_{ag}}{p_{eg} + \tau} = P_1^* < P_1 \\ \frac{\partial Y_{eg}}{\partial Y_{et}} &= -\frac{p_{et} + \tau}{p_{eg}} = P_2^* > P_2.\end{aligned}\quad (2)$$

The effect of the subsidy on the price/budget line is

$$\begin{aligned} I &\geq p_{ag} Y_{ag} + p_{et} Y_{et} + p_{eg} Y_{eg} \\ I &\geq p_{ag} Y_{ag} + (p_{et} + \tau) Y_{et} + p_{eg} Y_{eg} \\ I + \tau Y_{et} &\geq p_{ag} Y_{ag} + p_{et} Y_{et} + p_{eg} Y_{eg} \end{aligned} \tag{3}$$

In order to simplify the analysis, we focused on

- Agriculture/Food
- Manufactured Goods
- Energy
- Ethanol

Ethanol will be assumed to be an imperfect substitute for energy.
The inputs considered will be

- Capital
- Labor
- Energy (Primary energy)
- Materials
- Land

Again to simply the analysis (guarantee an interior point solution)
we used Constant Elasticity of Scale utility and production
functions

$$\begin{aligned}
 Y_j &= \left[\sum_i \alpha_{ij} x_{ij}^{\frac{1}{\sigma_j}} \right]^{\frac{\sigma_j}{\sigma_j-1}} \\
 U &= \left[\sum_i \beta_j^{\frac{1}{\phi}} Y_j^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}
 \end{aligned} \tag{4}$$

The resulting input and output demands are

$$\begin{aligned}
 X_{ij}(w, Y_j) &= \frac{Y_j \alpha_{ij}}{w_i^{\sigma_j} \left[\sum_k \alpha_{kj} w_k^{1-\sigma_j} \right]^{\frac{\sigma_j}{\sigma_j-1}}} \\
 Y_j(p, I) &= \frac{I \beta_j}{p_j^\phi \left[\sum_k \beta_k p_k^{1-\phi} \right]^{\frac{\phi}{\phi-1}}}
 \end{aligned} \tag{5}$$

The computable general equilibrium result is then found by solving the conditions for excess demand

$$\xi(w, p) = \begin{bmatrix} Y_j(p, I) - Y_j^* \\ \sum_j X_{ij}(w, Y_j^*) - E_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \forall i, j \quad (6)$$

First determine the demand for outputs

$$Y_j^* = Y_j(p^*, I) \quad (7)$$

Compute the excess demands for inputs

$$\xi_i(w^*, p^*) = \sum_j X_{ij}(w, Y_j^*) - E_i \quad (8)$$

To do this, our objective function is

$$\min_{w,p} \sum_i \xi_i (w, p)^2 \quad (9)$$

with an income closure constraint

$$\sum_i w_i E_i = I \quad (10)$$

a zero profit restriction

$$\sum_j p_j Y_j^* - \sum_i w_i E_i = 0 \quad (11)$$

and a price normalization equations

$$\sum_j p_j + \sum_i w_i = 9.43. \quad (12)$$

Table: Effect in Output Market

	1.00	1.18	Percent of Baseline
Energy Output	12.68394	12.88782	1.01607
Agricultural Output	2.35922	2.37640	1.00728
Manufacturing Output	38.98672	39.00774	1.00232
Ethanol Output	0.79464	0.79786	1.00405
Energy Price	1.42980	1.40080	0.97972
Agricultural Price	2.79179	2.74916	0.95797
Manufacturing Price	0.01841	0.01844	1.00163
Ethanol	0.95320	1.09750	1.15138
Returns to Factors	28.27616	28.14185	0.99525

Table: Change in Input Prices

	1.00	1.18	Percent of Baseline
Capital	0.23126	0.23069	0.99754
Labor	0.17447	0.17388	0.99662
Energy	1.42980	1.40080	0.97972
Materials	0.01841	0.01841	1.00163
Land	0.00109	0.00114	1.04587
Ag. Materials	2.79179	2.74916	0.98473

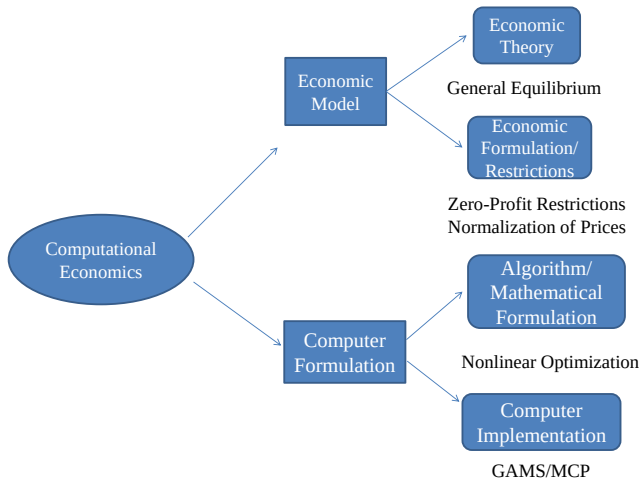


Figure: Subsidizing Ethanol Production

Table: Examples of Computational Economics

Factor	Investment under Irreversibility Uncertainty	Cash Basis versus Market Value Balance Sheets	Ethanol Computable General Equilibrium Model
Economic Theory	Option Pricing versus the Net Present Value Value of Investment	Accounting Theory versus the Financial Information in Accounting Statements	General Equilibrium, the Feedback of Prices through the Economy – Nonpositive Excess Demand
Economic Formulation/ Restriction	Option Pricing Theory – Merton	Generalized Accepted Accounting Principles	Arrow-Debreu: Price Normalization Zero Profit Conditions, etc.
Algorithm/ Mathematical Formulation	Integration by Simulated	Integration by Simulated Annealing	Solving Nonlinear Systems of Equations – Nonlinear Optimization
Computer	Excel/At Risk Simulation	Fortran – Simulation	GAMS/MCP