

Lecture II: Introduction to Computational Problems in Economics

Charles B. Moss ¹

¹University of Florida

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Numerical Integration

By this time most economic students have been introduced to usefulness of the integral. We use integrals to estimate producer and consumer surplus and in econometric estimators.

Mathematically, the standard approach to integrals is fairly straightforward for well behaved functions. In the case of the present value problem

$$\begin{aligned} V(t) &= \int_t^{-\infty} K \exp(-\delta s) ds = \lim_{a_1 \rightarrow \infty} \frac{K}{\delta} \exp(-\delta t) - \frac{a_1}{\delta} \exp(-\delta a_1) \\ &= \frac{K}{\delta} \exp(-\delta t). \end{aligned} \tag{1}$$

Numerical Integration – Continued

However, some integrals do not have analytical integrals. One such example is the integral of the normal distribution

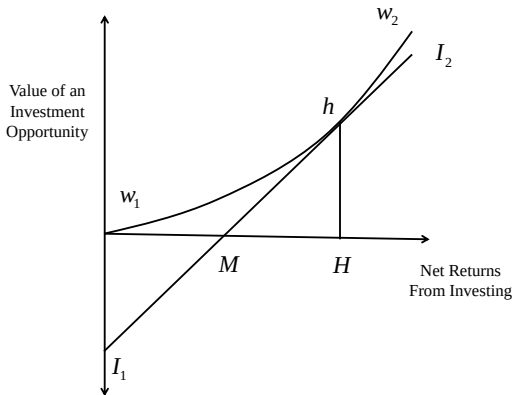
$$F(x \leq K) = \int_{-\infty}^K \frac{1}{\sqrt{2\pi}\sigma} \exp \left[-\frac{(x - \mu)^2}{2\sigma^2} \right] \quad (2)$$

does not exist. As a result, we have developed a number of ways to compute the value of the integral.

Investment Under Uncertainty and Irreversibility

Purvis et al. (1995) examines the effect of risk and irreversibility on an investment in an environmentally beneficial technology for dairy in Florida. The concept of irreversibility is that investments are largely sunk at the point that they are made. However, very few investment decisions are "now or never", but "now or later". Hence, there may be an advantage to waiting for more information to arrive. This formulation changes the investment decision into an option pricing model.

Investment Under Uncertainty and Irreversibility



Investment Under Uncertainty and Irreversibility

The mathematics of solving for this option value is based on the value of an investment following a geometric Brownian motion process

$$\frac{dV}{V} = \mu dt + \sigma dz. \quad (3)$$

where V is the value of the investment, μ is the average change in the value of the investment, σ is the standard deviation of the geometric process, and dz is a Wiener process (dz follows a normal distribution such that $E[dz] = 0$ and $E[(dz)^2] = dt$).

The computational component of this study involved the estimation of μ and σ *ex ante*.

Our assumption was that

$$V_{t,N} = \sum_{i=0}^N \frac{R_{t+i}}{(1+\rho)^i} \quad (4)$$

$$V_{t+1,N+1} = \sum_{i=1}^{N+1} \frac{R_{t+i}}{(1+\rho)^{i-1}}$$

such that $R_t \sim F(\mu_t, \sigma_t)$. Hence we could approximate

$$\begin{aligned} \mu &= E \left[\frac{dV}{V} \right] = E \left[\frac{\Delta(\ln V)}{\Delta t} \right] \\ &= E [\Delta(\ln V_t)] = E [\ln(V_t) - \ln(V_{t+1})] \end{aligned} \quad (5)$$

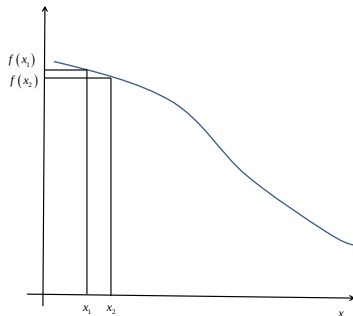
In theory, this expectation implies

$$E[\Delta \ln(V_t)] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left\{ \ln \left(\sum_{i=0}^N \frac{R_{t+i}}{(1+\rho)^i} \right) - \ln \left(\sum_{i=1}^{N+1} \frac{R_{t+i}}{(1+\rho)^{i-1}} \right) \right\} dF(R_t, R_{t+1}, \dots, R_{t+N+1}) \quad (6)$$

with a similar expectation defining σ .

Simpson's Rule

There are several ways to numerically integrate a complicated function like Equation 6. One procedure is called Simpson's rule. Using the function in Figure 2 we could define the area under the first slice of the function as



$$\Delta_1 = (x_2 - x_1) \times f(x_2) + \frac{1}{2} (f(x_1) - f(x_2)) (x_2 - x_2) \quad (7)$$

Thus, chopping the entire range of the integral into pieces

$$\int_{x_1}^{x_N} f(x) dx \approx \sum_i^N \Delta_i \quad (8)$$

Making the slices progressively smaller yields a better approximation of the integral.

Simulated Annealing

A second approach is based on concepts of sampling theory. Remembering the concepts of a theoretical expectation and a sample expectation from statistics

$$\mu = E(x) = \int_{-\infty}^{\infty} xf(x) dx \Leftrightarrow \frac{1}{N} \sum_{i=1}^N x_i \quad (9)$$

As depicted in Figure 3, sampling or randomly drawing from the known distribution provides an integral.

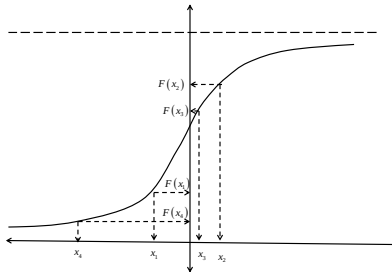


Figure: Sample Theory

Purvis et al. (1995) uses the later approach to derive estimates of μ and σ .

Cash Versus Market Value Balance Sheets

Moss, Klinefelter and Gunderson (2012) discuss four primary users of accounting information:

1. Individuals involved in ownership transactions whose data requirements are the subject of GAAP manifested currently through the Financial Accounting Standard Board (FASB).
2. Users of accounting information internal to the firm historically referred to as cost accounting, more recently referred to as managerial accounting (Demski, 2008).
3. Taxation or the collection of government revenue. This use is codified in the U.S. Tax Code.
4. Regulatory users such as government oversight of utilities.

Financial accounting, the primary dialect, allows lenders and investors to access the amount, timing, and certainty of a corporations future cash flows. Creditors want to know if theyll get their money back; stock investors care about whether they can expect substantial future dividends. Financial accounting principles merged to match revenues with expenses and determines a corporations ability to pay interest or dividends from business activity in a given period (King, 2006, p. 4).

Most farm businesses in the US fit somewhat irregularly into this mold. The ownership structure of commercial agriculture in the US is largely sole proprietorships with a few closely held partnerships and corporations. Thus, the ability to adequately represent earnings potential (or earnings per share) to possible investors who would purchase newly issued stock or for the enrichment of current owners of common stock is not important for the majority of farms in 2012. (However, there may be a scale bias with larger farms increasingly interested in this dimension).

The ownership aspect of interest under the current debt structure is the credit market. Thus, financial statements are important to farmers who want to obtain a mortgage to purchase land, borrow to purchase equipment, or obtain an operating loan to put a crop in the ground. In this transaction, the farmer represents his credit-worthiness to the potential lender or a potential outside equity investor to give evidence of repayment capacity (Moss, Klinefelter, and Gunderson, 2012).

The relevant question is: How much information does traditional cost-basis balance sheets and income statements contain compared with market valued balance sheets? As a starting point consider what the objective of the accounting information depicted in Figure 4. Assume that we want to develop a measure $\mu_t(\Omega_t)$ which informs decision makers about the probability that the firm will be able to pay its fixed obligations in the next period. From an informational context, we are interested in minimizing the information inequality

$$I[f(\Omega_t), g(\mu_t(\Omega_t))] = \int_{-\infty}^{\infty} f(\Omega_t) \ln \left[\frac{f(\Omega_t)}{g(\mu_t(\Omega_t))} \right] d\Omega_t \quad (10)$$

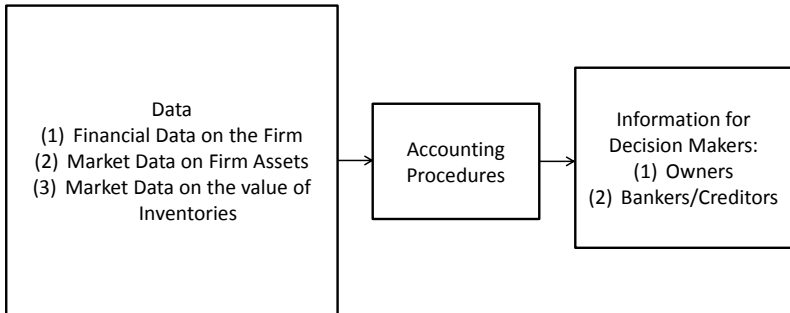


Table 8 presents the land values, rates of return, rate of return on assets and implied interest rates for florida agriculture. To start to unravel the cash versus market value information, I assume that each firm starts with 1,280 acres of land in 2003 and then randomly purchases another 1,280 acres before 2012. Thus, every firm has 2,560 acres in 2012 which are is valued as \$11,775,000, but the cash-based assets for each firm are presented in Table 9.

Florida Land Values

Year	Land Values	Rate of Change	Rate of Return on Assets	Implied Interest Rate
2012	4600	-0.0215		
2011	4700	-0.0211		
2010	4800	-0.0704	0.0464	0.0675
2009	5150	-0.0909	0.0630	0.0580
2008	5640	0.0251	0.0393	0.0646
2007	5500	0.0333	0.0410	0.0615
2006	5320	0.1049	0.0264	0.0692
2005	4790	0.4351	0.0217	0.0775
2004	3100	0.0667	0.0276	0.0608
2003	2900	0.0641	0.0655	0.0921
2002	2720	0.0451		
2001	2600	0.0800		
2000	2400	0.0601		
1999	2260	0.0089		
1998	2240	0.0180		
1997	2200	0.0230		
1996	2150			

Cash Balance Sheet

Firm	Cash Based Balance Sheet	Cash Based Rate of Return					Average Return
		1	2	3	4	5	
1	10,099,103	8.31	7.04	6.30	5.79	5.72	6.63
2	10,567,786	7.94	6.72	6.02	5.53	5.46	6.33
3	9,887,552	8.49	7.19	6.43	5.91	5.84	6.77
4	9,957,637	8.43	7.14	6.39	5.87	5.80	6.73
5	10,043,286	8.36	7.07	6.33	5.82	5.75	6.67
6	10,043,286	8.36	7.07	6.33	5.82	5.75	6.67
7	9,959,904	8.43	7.13	6.38	5.87	5.80	6.72
8	10,320,144	8.13	6.88	6.16	5.66	5.59	6.48
9	10,377,050	8.09	6.85	6.13	5.63	5.56	6.45
10	10,075,777	8.33	7.05	6.31	5.80	5.73	6.64
Market	11,776,000	7.13	6.03	5.40	4.96	4.90	5.68

Defining the rate of return as

$$\frac{dV(t)}{V(t)} = r(t) - k(t) + \nu(t) \Leftrightarrow \begin{cases} V(t) = \int_t^{\infty} R(s) e^{-k(s)} ds \\ r(t) = \frac{R(t)}{V(t)} \end{cases} \quad (11)$$

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \quad (12)$$

$$r_{it} = \alpha_{0i} + \alpha_{1i} r_{i,t-1} + (\beta_i \epsilon_t + \tilde{\epsilon}_{it}) \ni \frac{1}{N} \sum_{i=1}^N \beta_i = 1. \quad (13)$$