

Lecture VII: Numeric Integrals - Historical Cost versus Market Value Accounting

Charles B. Moss

September 6, 2012

1 Predictive Use of Financial Statements in Making a Loan

As a simple example, assume that a farmer applies for a loan with a commercial bank. Typically, the banker and farmer attempt to estimate his income and equity position (as discussed in Chapter 3.6 most farmers do not construct financial statements on a regular periodic basis because of the cost). Intuitively, the banker and farmer could construct these financial statements using either accrual accounting system relying on historical cost (psuedo Generally Accepted Accounting Principles [GAAP] compliant) or market valued principles. First, it should be noted that the information would probably not be available for the construction of fully GAAP compliant statements. The matching of cash expenditures on crops with each crop's disposition (i.e., either sales leading to the recognition of income or inventory leading to the cost based recognition on the balance sheet) would be a daunting task for the smallest of farms. Second, even if those statements could be constructed, it is highly unlikely that the statements could ever reach the standards of being auditable. Auditing requires the establishment of procedures for recognizing accounting events. The audit is usually a statement that the firm's procedures are being followed (i.e., the auditor checks the procedure for recognizing when expenses have occurred) and that the firm uses the results of this procedure in a way that is consistent with GAAP. Thus, the farmer and lender are more likely to construct a market valued set of financial statements.

The relevant question is then which accounting statements are more useful? If the accrual accounting statements based on historical cost are more

useful, then the question would be whether the value added by GAAP compliant statements exceed the cost of maintaining such systems at the farm level. Alternatively, it is possible that the market valued financial statements are more useful.

For our purpose here, assume that the banker is interested in whether the farmer would be able to repay the principal and interest of an operating loan (P) at the end of the period from his income from agricultural operations (R) given his consumption requirements (C). Mathematically, the farmer would be able to pay the loan if

$$R - P - C \geq 0 \Rightarrow R \geq P + C. \quad (1)$$

Recognizing that income is a random variable, the lender will make the loan if

$$P[R \geq P + C] \geq \delta \quad (2)$$

where $P[\cdot]$ denotes the probability and δ is some level of probability set by the bank policy (say $\delta = 0.95$). Defining $f[R_i]$ as the probability distribution function (i.e., the probability that $R = R_i$), the probability that the farmer will be able to make the loan payment can be defined as

$$P[R \geq P + C] = \sum_{i \in R_i \geq P+C} f[R_i]. \quad (3)$$

Intuitively, this probability is a function of other factors. For example, the probability of obtaining a certain income this year (R_t) may be a function of income last year (R_{t-1}), the level of assets last year (or better yet at the end of the last year/beginning of this year V_{t-1}), other debts owed by the farmer at the beginning of the year (D_{t-1}), etc. Thus, we can rewrite the probability distribution function as

$$f[R_i] \Rightarrow f[R_{it} | R_{t-1}, V_{t-1}, D_{t-1}, a_t] \quad (4)$$

where the vertical line ($|$) signifies such that (or given that we know) and a_t is a host of other factors not included in the financial statements. Thus, having information on these factors (i.e., R_{t-1} , V_{t-1} , D_{t-1} , a_t largely from the financial statements) allows the lender to refine the probability that the loan will be repaid

$$P[R_t \geq P + C | R_{t-1}, V_{t-1}, D_{t-1}, a_{t-1}] = \sum_{i \in R_t \geq P+C} f[R_{it} | R_{t-1}, V_{t-1}, D_{t-1}, a_t] \geq \delta. \quad (5)$$

The point of the accrual/historical cost versus market value financial statements then involves the specification of the probability function. Specifically, assume that we have two probability distribution functions $f[R_{it} | R_{t-1}, V_{t-1}, D_{t-1}, a_t]$ based on the market value financial statements and $g[\tilde{R}_{it} | \tilde{R}_{t-1}, \tilde{V}_{t-1}, \tilde{D}_{t-1}, a_t]$ based on historical cost financial statements. In this example the probability of market valued income R_{it} is a function of market valued lagged returns R_{t-1} , market valued assets V_{t-1} and market valued debt levels D_{t-1} while the probability of accrual/historical valued returns $\tilde{R}_{it}$ is a function of lagged accrual/historical based returns $\tilde{R}_{t-1}$, historically valued assets $\tilde{V}_{t-1}$, and historically valued debts $\tilde{D}_{t-1}$. The least difference will probably occur in the valuation of debt which is fixed by contract (assume D_{t-1}^* is the contractual debt level common to both probability distribution functions). The two specifications may yield different expectations of the probability of repayment

$$P_M[R_t \geq C + P | R_{t-1}, V_{t-1}, D_{t-1}^*, a_t] = \sum_{i \in R_{it} \geq C+P} f[R_{it} | R_{t-1}, V_{t-1}, D_{t-1}^*, a_t] = \delta_M$$

$$P_C[\tilde{R}_t \geq C + P | \tilde{R}_{t-1}, \tilde{V}_{t-1}, D_{t-1}^*, a_t] = \sum_{i \in \tilde{R}_{it} \geq C+P} g[\tilde{R}_{it} | \tilde{R}_{t-1}, \tilde{V}_{t-1}, D_{t-1}^*, a_t] = \delta_C \quad (6)$$

The point here is that there is a true probability of repayment that we estimate with two different procedures. The question is really which estimation is more reliable or more useful?

To make the question more concrete, let us assume that we have a sequence of cash/accrual and market based financial statements ($\{ \{ \tilde{R}_{2010}, \tilde{V}_{2010} \}, \{ \tilde{R}_{2009}, \tilde{V}_{2009} \} \cdots \{ \tilde{R}_{2003}, \tilde{V}_{2003} \} \}$ being a sequence of cash/accrual based returns and asset levels, and $\{ \{ R_{2010}, V_{2010} \}, \{ R_{2009}, V_{2009} \} \cdots \{ R_{2003}, V_{2003} \} \}$ being a sequence of market based returns and asset levels). Assuming that these returns are normally distributed, we could estimate the parameters of two distributions, one based on cash/accrual data and the other based on market value data

$$\begin{aligned}\bar{R}_C &= \frac{1}{8} \sum_{t=2003}^{2010} \tilde{R}_t, \sigma_C^2 = \frac{1}{8} \sum_{t=2003}^{2010} (\tilde{R}_t - \bar{R}_C)^2 \\ \bar{R}_M &= \frac{1}{8} \sum_{t=2003}^{2010} R_t, \sigma_M^2 = \frac{1}{8} \sum_{t=2003}^{2010} (R_t - \bar{R}_M)^2\end{aligned}\tag{7}$$

where $\bar{R}_C$ is the cash/accrual return, σ_C^2 is the variance of the cash/accrual returns, $\bar{R}_M$ is the average market based return, and σ_M^2 is the variance of those returns. Like the standard statistical t-test, we can estimate the probability that the farm will be able to make its loan payments under each estimate as

$$\begin{aligned}\hat{\delta}_C &= \Phi \left(\frac{\bar{R}_C - [C + P]}{\sqrt{\sigma_C^2}} \right) \\ \hat{\delta}_M &= \Phi \left(\frac{\bar{R}_M - [C + P]}{\sqrt{\sigma_M^2}} \right)\end{aligned}\tag{8}$$

where Φ is the cumulative distribution for the standard normal distribution.

Taking the Agricultural Resource Management Survey's income statements for Florida for 2003 through 2010 presented in Table 1 as an example, the expected return is \$ 34,254 with a standard deviation of \$ 12,431. Adding the current debt, the notes payable within one year, current portion of the long term debt, accrued interest, and accounts payable for 2010 yields \$ 13,767 of debt plus principal to be paid in the current year. If we assume that the farmer will need \$ 15,000 for consumption in the current year, $P + C = 28,757$. Hence, the probability of making these cash commitments out of current income can be computed to be

$$\hat{\delta} = \Phi \left[\frac{34,254 - 28,757}{13,767} \right] = \Phi [0.4414] = 0.67.\tag{9}$$

Thus, using the USDA's ARMS financial statements (which are probably closer to market valued than accrual/cash based financial statements) the probability that the average Florida farm will be able to cash flow its liabilities in 2011 is 67 percent.

Table 1: Agricultural Resource Management Survey – Income Statement for Florida

	2010	2009	2008	2007	2006	2005	2004	2003
Gross Income	156,107	162,935	167,679	146,226	143,752	126,859	118,975	120,822
Variable Cost	99,595	97,707	117,284	104,201	100,906	93,902	89,835	90,684
Fixed Cost								
Interest	3,688	3,541	3,855	4,228	3,048	2,651	2,010	5,158
Insurance	3,092	2,069	3,081	3,466	2,490	2,618	2,501	2,426
Rents and Lease	2,836	4,310	2,441	4,926	3,899	3,215	2,253	1,410
Net Cash Income	46,896	55,308	41,018	29,405	33,409	24,473	22,376	21,144
Depreciation	6,603	7,267	5,566	5,507	4,904	4,057	6,419	6,546
Net Income	40,293	48,041	35,452	23,898	28,505	20,416	15,957	14,598

2 Simulated Annealing Model

We are interested in working with three equations. First, we hypothesize that returns follow a random walk process

$$r_{it} = \alpha_i + \beta_i \xi_t + \xi_{it} \quad (10)$$

where r_{it} is the rate of return on agricultural assets for firm i at time t , α_i is firm specific mean for firm i , β_i is a single index relationship between the randomness in the return for firm i and the randomness of the overall market, ξ_t is the common randomness in the overall market, and ξ_{it} is the randomness specific to firm i . The overall market return is then formulated as

$$r_t = \alpha + \xi_t \ni \alpha = \frac{1}{N} \sum_{i=1}^N \alpha_{it}. \quad (11)$$

Given the overall market return, we assume that asset values follow a geometric Brownian motion process

$$\frac{dV}{V} = r_t - k_t + \nu_t \quad (12)$$

where V is the value of farmland in dollars per acre, r_t is the observed average return, k_t is a cost of capital, and ν_t is a random normal variate. The final equation is then the consumption equation from Ramirez, Moss, and Boggess (1997)

$$C_t = E_t \left[\frac{\rho - k_t b}{(1 - b)} - \frac{b (k_t - \mu_{at})^2}{2 (1 - b) \sigma_{At}^2} \right] \quad (13)$$

where ρ is the decision maker's rate of time preference, b is the decision maker's risk aversion coefficient, μ_{at} is the decision maker's expected return on assets (we will set this equal to α_i), and σ_{At}^2 is the decision maker's variance on asset returns (we will set this equal to $\beta_i^2 \sigma^2 + \sigma_i^2$ or the sum of the variance from the common factor and the individual's idiosyncratic variance). These data can be defined for Florida using the ARMS data for 2003 through 2010 as presented in Table 2. By assumption, we will allocate one half of the variance of returns to the common variation and one half to idiosyncratic risk. The risk aversion coefficient is solved by inverting the optimal debt

Table 2: Florida Data

Year	Return	Asset Values	Rate of Return	Debt to Asset	Risk Aversion (b)	Land Values	Net Change
2002						2,720	
2003	27,712	668,179	0.04147	0.09042	-85.356	2,900	0.04260
2004	26,639	1,102,502	0.02416	0.03065	-91.031	3,100	0.06253
2005	30,339	1,666,244	0.01821	0.02151	-91.898	4,790	0.43692
2006	40,356	1,707,458	0.02364	0.02592	-91.480	5,320	0.10131
2007	38,559	1,333,905	0.02891	0.05052	-89.145	5,500	0.02437
2008	47,314	1,497,129	0.03160	0.03439	-90.676	5,640	0.01353
2009	63,159	1,184,860	0.05331	0.04706	-89.473	5,150	-0.12419
2010	53,420	1,348,258	0.03962	0.04301	-89.858	4,800	-0.09000
Mean	40,937	1,313,567	0.03261	0.04293	-89.865	4650.000	0.05838
Std Dev	13,027	336,470	0.01153	0.02172	2.062	1062.921	0.17068

position. Ramirez, Moss, and Boggess derive the optimal debt-to-asset ratio to be

$$\delta_t^* = 1 - \frac{(1-b)\sigma_{At}^2}{\mu_{At} - k_t} \quad (14)$$

solving this expression for b yields

$$b = 1 - \frac{(1-\delta)(\mu_{At} - k_t)}{\sigma_{At}^2} \quad (15)$$

where $k_t = 0.02$.

2.1 Procedure

To construct the farm level results, we start by drawing different farms. To do this we draw α_i , β_i and b_i (for our purpose here we will set $\rho = 0.03$ and $k_t = 0.02$). The data presented in Table 2 provide guidance for the distributions of α_i and b_i . Since we want idiosyncratic risk to cancel out, we want the average β_i to be 1. As a starting point, we assume the variance of β_i to be 0.15. Under our baseline results, $C_t = 0.0266$ which implies a consumption of \$ 33,475 at the average equity position. How do we draw these

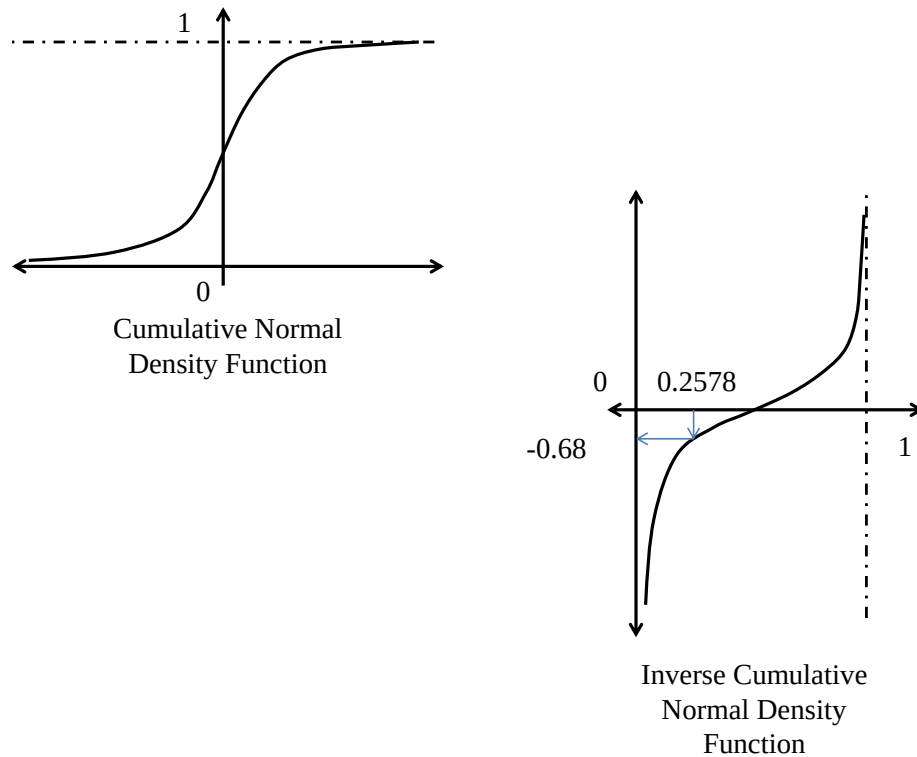


Figure 1: Inverse Function Formulation

values. The basic idea behind Simulated Annealing is drawing random numbers. In this case, we want to draw 250 normals (not necessarily correlated). Most computer programs have uniform random number generators (Excel and Fortran). They do not have normal random number generators. There are several approaches to generating normal random numbers, we will use the inverse function formulation. We will draw a uniform random variable in Fortran and then determine the cumulative value of the normal that would yield that probability. The procedure is graphically depicted in Figure 1

Table 3 presents a synopsis of the individual firms. Using the firms, we can then draw 25 years of data for each of the variables. As a starting point, consider the results for income and farmland values. As a first step, we draw the common component of returns

Table 3: Random Firm Draws

Draw	α_i	β_i	b_i
1	0.29383489E-01	0.10039910E+01	-0.90079188E+02
2	0.32846847E-01	0.99552833E+00	-0.93317930E+02
3	0.33949277E-01	0.10026855E+01	-0.86748064E+02
4	0.31755884E-01	0.10002811E+01	-0.87889719E+02
5	0.27370927E-01	0.99070585E+00	-0.89711379E+02
6	0.33887501E-01	0.98602911E+00	-0.90476460E+02
7	0.28287182E-01	0.99879949E+00	-0.88861260E+02
8	0.33258019E-01	0.10085255E+01	-0.87761183E+02
9	0.32804774E-01	0.99233849E+00	-0.89048453E+02
$\vdots$	$\vdots$	$\vdots$	$\vdots$
250	0.30678922E-01	0.10022455E+01	-0.95202366E+02
Mean	0.0327	0.9996	-89.9562
Std. Dev.	0.0030	0.0054	1.9959

$$\xi_t = (0.25 \times 0.01153) \epsilon_{1t} \quad (16)$$

where ϵ_{1t} is a standard normal distribution. Next, we draw each individual firm's return

$$r_{it} = \alpha_i + \beta_i \times \xi_t + (0.75 \times 0.01153) \epsilon_{2t} \quad (17)$$

where ϵ_{2t} is another standard normal variable. Next, we average the individual returns to the aggregate return

$$r_t = \frac{1}{250} \sum_{i=1}^2 50r_{it}. \quad (18)$$

This average return is used to generate the aggregate change in land values

$$\frac{\Delta V_i}{V_i} = (r_t - 0.02) + 0.007\nu_t \quad (19)$$

where ν_t is the standard normal. This change in land values then gives the actual land values

$$V_t = V_{t-1} \times \left(1 + \frac{\Delta V_t}{V_t}\right) \quad (20)$$