

# Lecture I: Computational Versus Analytical Economics

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As a starting point, let's think about what you have typically done in your theory classes up to this point. Take the derivation of producer behavior

$$\begin{aligned} \max_x & p'y - w'x \\ \text{s.t. } & \{y, x\} \in T \end{aligned} \tag{1}$$

where  $y$  is a vector of outputs,  $p$  is a vector of output prices,  $x$  is a vector of input levels, and  $w$  is a vector of input prices. In order to be a little more concrete, let's replace the production technology with a Cobb-Douglas production function of a single output with three inputs

$$\max_x px_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3} - w_1 x_1 - w_2 x_2 - w_3 x_3 \tag{2}$$

Given this formulation, we are interested in the economic behavior embodied in this formulation.

## Q1 Does a solution exist?

- Demonstrate that a solution obeys the first and second order necessary conditions.
- Demonstrate under what conditions the problem can obey the first and second order necessary conditions.

## Q2 Is that solution Unique?

- Demonstrate first and second order sufficient conditions.

Q3 As the point of the solution can you say something about the implied behavior – Demand curves slope downward.

- In both of these pursuits the formulation in Equation 2 allows for a closed form or analytical solution.
  - Pages 15-17 of my production book demonstrates the derivation of the cost function for the two-input Cobb-Douglas production function

$$\begin{aligned}
 x_1^*(w_1, w_2, y) &= y^{\frac{1}{\alpha + \beta}} \left( \frac{\alpha w_2}{\beta w_1} \right)^{\frac{\beta}{\alpha + \beta}} \\
 C(w_1, w_2, y) &= y^{\frac{1}{\alpha + \beta}} w_1^{\frac{\alpha}{\alpha + \beta}} w_2^{\frac{\beta}{\alpha + \beta}} \left[ \left( \frac{\alpha}{\beta} \right)^{\frac{\beta}{\alpha + \beta}} + \left( \frac{\beta}{\alpha} \right)^{\frac{\alpha}{\alpha + \beta}} \right] \quad (3)
 \end{aligned}$$

- We can take the derivative of the derived demand functions and demonstrate that they slope downward in own input price.
- We can take the derivative of the cost function with respect to the level of output to demonstrate that cost is increasing at an increasing rate.
- The point here is what if the production function has no closed form solution?

$$f(x_1, x_2) = \ln(\alpha_1 x_1 + \alpha_2 x_2 + A_{12} x_1 x_2) + \beta_1 x_1 + \beta_2 x_2 + B_{12} x_1 x_2 \quad (4)$$

- Can we solve for this optimal numerically and then use other numerical methods to describe the optimal behavior?