

Aggregation of Welfare Analysis: Lecture XVIII

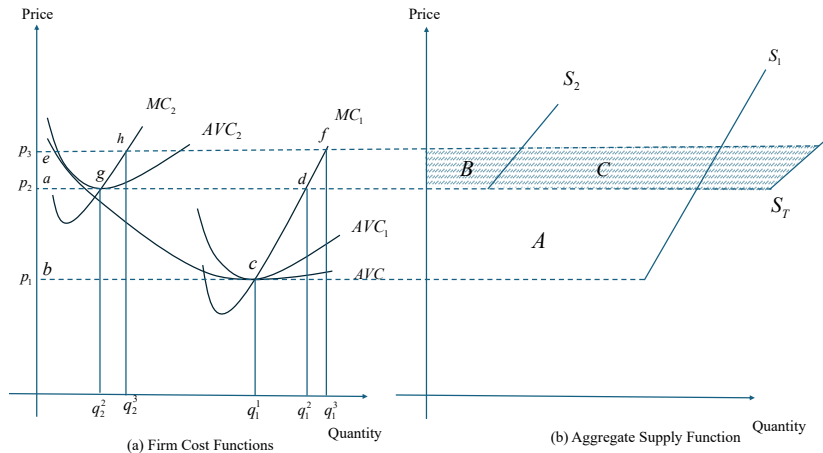
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Aggregation of WTP: The Producer Case

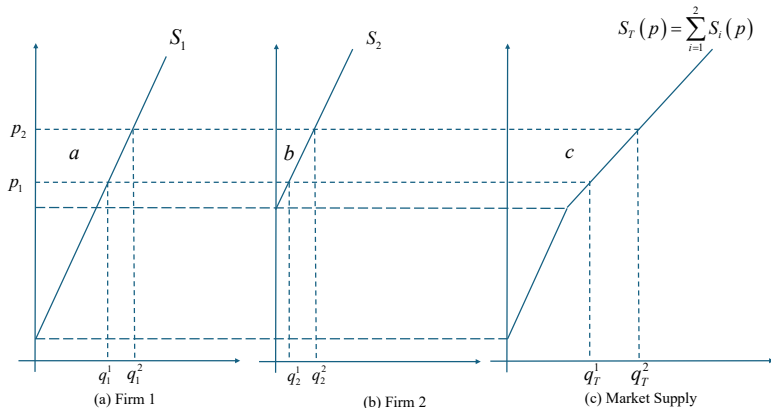
- As a starting point, consider our welfare measures as applied to the producer.
- Figure 4 presents the cost functions for two firms – we are assuming that firm 1 has more of some factor of production than firm 2. Implicitly, firm enters production at a lower price than firm 2.
- At a price of p_1 , only firm 1 produces and the quantity produced is q_1^1 .
- At a price of p_2 firm 2 will start to produce and initially produce a quantity of q_2^2 .
- The increase in the price from p_1 to p_2 yields an increase in the Producer Surplus of $abcd$ (focusing for a moment on panel (a)).



- As the price increases from p_2 to p_3 , firm 1's production increases from q_1^2 to q_1^3 and the welfare increases by $dadf$.
- Firm 2's production increases from q_2^2 to q_2^3 and gains producer welfare of $eagh$.
- Note we can derive the same measures in Panel (b) (the Aggregate Supply Function).
 - The area $abcd$ in panel (a) and area A in panel (b).
 - The area $eadf$ in panel (a) is $B + C$ in panel (b).
 - The area $eagh$ in Panel (a) is B in panel (b).
 - Note that the total change in producer WTP for the price increase from p_2 to p_3 is $C + 2B$.
- The trivial mathematics are

$$S_T(p) = \sum_{i=1}^N S_i(p)$$

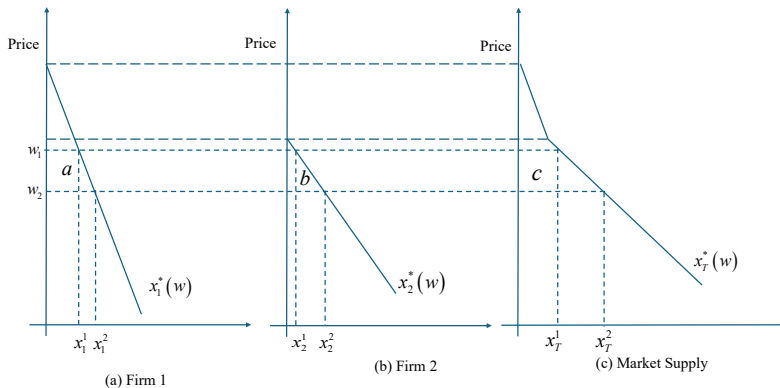
$$\int_{p_2}^{p_3} S_T(p) dp = \sum_{i=1}^N \int_{p_2}^{p_3} S_i(p) dp \quad (1)$$

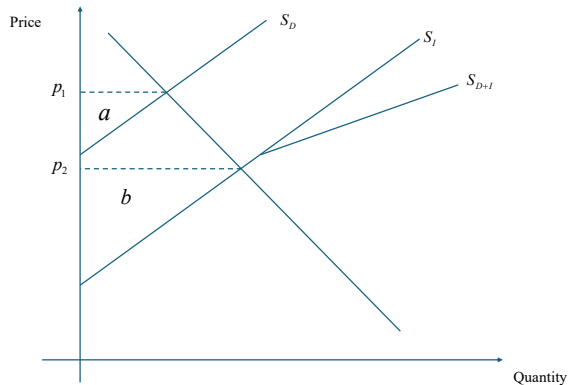


- Keeping our focus on the producers, the derived demand curves largely follow the derivations for supply

$$\begin{aligned}x_T(w) &= \sum_{i=1}^N x_i(w) \\ \int_{w_1}^{w_2} x_T(w) dw &= \sum_{i=1}^N \int_{w_1}^{w_2} x_i(w) dw\end{aligned}\tag{2}$$

- So the graphical derivation for the derived demand curve as presented in Figure 8 largely follows the general intuition of the supply functions from Figure 6.
- Specifically, if the wage rate falls from w_1 to w_2 , firm 1's gain in surplus is a and firms 2's is b . In total, all producers gain $c = a + b$.



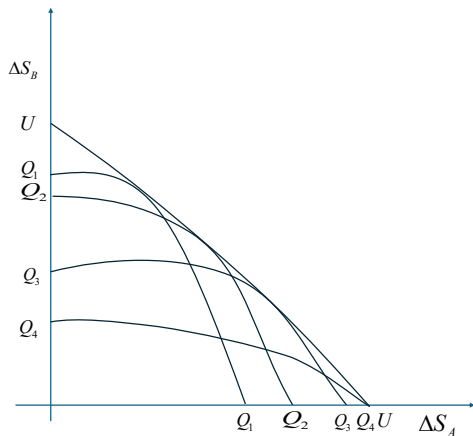


Aggregation of WTP: The Case of Consumers and Factor Owners

- In the forgoing examples, we are fairly comfortable with the use of profit, cost, and marginal value product to derive our welfare measures.
- The question is – can we apply our standard surplus measures consumer demand and factor supply?
 - In the preceding lectures, we have developed the difference between these surplus measures and WTP.
 - In both cases the divergence between the standard surplus measure has involved the 'income effect.'
 - We know that $C \leq \Delta S \leq E$, we know that we can bound these effects if $|\eta_j^* s_j^*| < 0.05$ $0.95\Delta S \leq WTP \leq 1.05\Delta S$.
 - Thus, the market change in consumer surplus is no more than 5 percent in error if corresponding conditions are met that limit error to 5 percent for all individuals.

Aggregation of Money Measure of Utility Change

- Recalling some of our initial discussions – early welfare economists sought a unique money measure of utility change; however, such a measure required us to add these money measure of utility change across individuals.
 - Just, Hueth, and Schmitz, discuss adding $U_1(q)$ with $U_2(q) = 2U_1(q)$ where both utility functions yield the same optimal amount consumed, but different utilities.
- The approach we use in empirical welfare analysis follows Samuelson's utility possibilities approach.
 - Specifically, we assume that a unique money measure of utility change with respect to a price change exists and can be measured for each individual effected by the price change.
 - Because each of these measured effects are consistent with each individual's ordinal preferences – we miss the $U_2(q) = 2U_1(q)$ debacle.

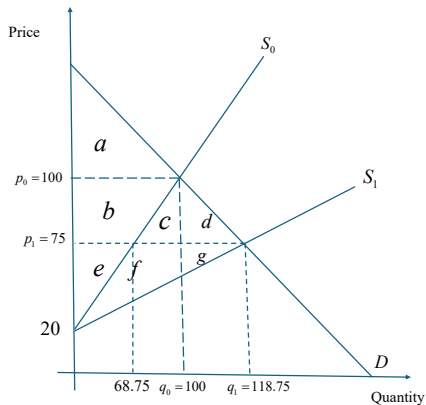


Aggregation of WTP Over Producers and Consumers

- To bring the two parts together, consider the scenario where either the federal or a state-level government investments money in agricultural research - who are the gainers and/or losers?
- As depicted in Figure 14, the general concept is that the agricultural supply curve rotates outward from S_0 to S_1 .
 - We will assume that the original equilibrium is where $p_0 = 100$ and $q_0 = 100$. We will assume the elasticity of demand is -0.75 while the elasticity of supply is 1.25 which yields

$$\begin{aligned}q_D &= 175 - 0.75p \\ q_S &= -25 + 1.25p\end{aligned}\tag{3}$$

- Next, we assume that the rotation of the supply curve yields an equilibrium price of $p_1 = 75$, at this equilibrium the consumers will demand $q_1 = 118.75$.



- Using the same point of origination for the supply curve ($\tilde{p} = 20$), the slope of the new supply function is 1.9167 (which implies a slightly small elasticity of supply $\tilde{\eta}_S = 1.2105$)

$$\tilde{q}_S = -25 + 1.9167p. \quad (4)$$

- Working on the consumers first - the gain to consumer surplus is

$$\begin{aligned} b + c + d &= 100 \times (100 - 75) + (118.75 - 100) \times (100 - 75) / 2 \\ &= 2,734.38 \end{aligned} \quad (5)$$

- The gain/loss to producers is somewhat more complicated - First producers will lose the area b , but gain the area $f + g$.

- Starting with v

$$\begin{aligned} b &= 68.75 \times (100 - 75) + (100 - 68.75) \times (100 - 75) / 2 \\ &= 2,109.38 \end{aligned} \quad (6)$$

- Next, the 'gain' can be written as $f + g$, but it is easier to compute $e + f + g$

$$\begin{aligned} e + f + g &= (75 - 20) \times (118.75 - 0) / 2 = 3,265.63 \\ e &= (75 - 20) \times (68.75 - 0) / 2 = 1890.625 \\ \therefore f + g &= 3,265.63 - 1,890.63 = 1,375.00 \end{aligned} \quad (7)$$

- Therefore, the producers lose $1,375.00 - 2,109.38 = -734.38$.
In net the economy gains 2,000.

- For future consideration - what did it cost to gain this surplus? Suppose that the government spent 10,000 on the research program.

$$PV = -10,000 + \sum_{i=1}^{10} \beta^i 2,000 \quad (8)$$

where β is a social discount rate. At say $\beta = 1/1.05$
 $PV = -10,000 + 15,443.47 = 5,443.47$.

Benefit/Cost Analysis

- Another way that Welfare Analysis is posed is Benefit/Cost Analysis – where the present value of benefits is divided by the present value of costs

$$\frac{\sum_{i=1}^{10} \frac{1}{(1+r)^i} 2,000}{10,000} = \frac{2,000 \left(\frac{1 - (1 + 0.05)^{-10}}{0.05} \right)}{10,000} \quad (9)$$

$$= \frac{15,443.47}{10,000} = 1.54443$$

- So the default is that the benefits exceed the costs if the B/C ratio is greater than 1.0. This may be used to “rank” policy alternatives, especially when the policy requires some “investment” from the government that would pay out over time.