

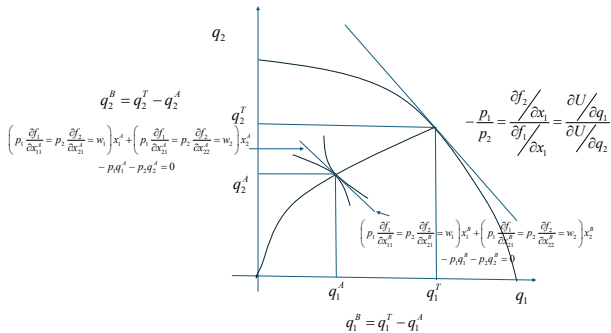
General Solution of the Social Welfare Problem: Lecture XIIIb

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1 Initial Solution

Initial Solution



Consider Adding a Subsidy for the Production of q_1

- First, we will call the original solution (q_1, q_2, p_1, p_2) which satisfy the conditions

$$-\frac{p_1}{p_2} = \frac{\frac{\partial f_2}{\partial x_1}}{\frac{\partial f_2}{\partial x_2}} = \frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}} \quad (1)$$

which gives us the budget constraint in the Edgeworth box

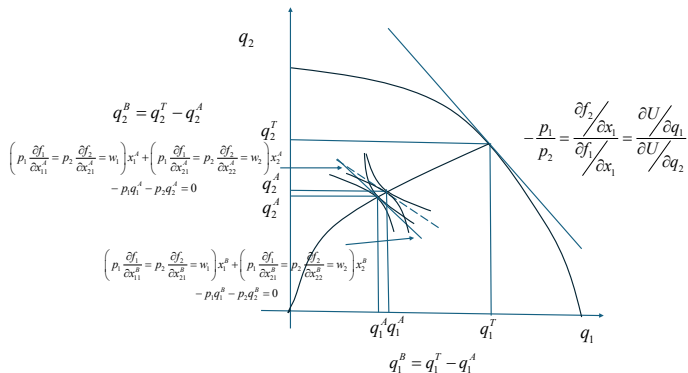
$$\left(p_1 \frac{\partial f_1}{\partial x_{11}^A} = p_2 \frac{\partial f_2}{\partial x_{21}^A} \right) x_1^A + \left(p_1 \frac{\partial f_1}{\partial x_{21}^A} = p_2 \frac{\partial f_2}{\partial x_{22}^A} \right) x_2^A \quad (2)$$
$$-p_1 q_1^A - p_2 q_2^A = 0$$

- Next, I am going to hypothesize a “subsidy” payment on the production of q_1 of τ . We will call this solution $(\tilde{q}_1, \tilde{q}_2, \tilde{p}_1, \tilde{p}_2)$ which solves a slightly different problem

$$\pi = (p_1 + \tau) f_1(x_{11}, x_{21}) + p_2 f_2(x_{12}, x_{22}) - w_1(x_{11} + x_{21}) - w_2(x_{12} + x_{22}) \quad (3)$$

Going to a modified version of Equation 2

$$\begin{aligned} \left[(\tilde{p}_1 + \tau) \frac{\partial f_1}{\partial x_{11}^A} \Big|_{x_{11}^A = \tilde{x}_{11}^A} = \tilde{p}_2 \frac{\partial f_2}{\partial x_{21}^1} \Big|_{x_{21}^A = \tilde{x}_{21}^A} \right] x_1^A + \\ \left[(\tilde{p}_1 + \tau) \frac{\partial f_1}{\partial x_{12}^A} \Big|_{x_{12}^A = \tilde{x}_{12}^A} = \tilde{p}_2 \frac{\partial f_2}{\partial x_{22}^1} \Big|_{x_{22}^A = \tilde{x}_{22}^A} \right] x_2^A + \\ -\tilde{p}_1 \tilde{q}_1 - \tilde{p}_2 \tilde{q}_2 = 0 \end{aligned} \quad (4)$$



- This subsidy causes the solution to shift to another point on the contract curve based on the ownership of the factors of production. In the current case, we assume that factors ownership “favors” individual A (i.e., individual A owns relatively more of the factor that is used to produce the first good.)
- Note that we would have to account for the money required to make the payment

$$\tilde{m}_i = \tau \left(\frac{\partial f_1}{\partial x_{11}^A} x_1^A + \frac{\partial f_1}{\partial x_{12}^A} x_2^A \right) \quad (5)$$