

Willingness-to-Pay – Marshallian Versus Hicksian Demand: Lecture XIII

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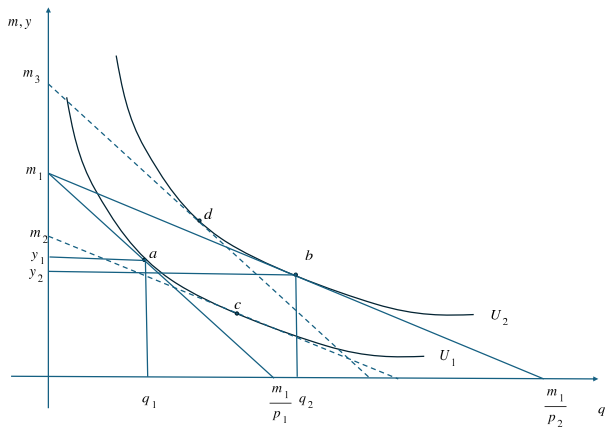
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1 Willingness-To-Pay Measures

Willingness-To-Pay Measures

- In Lectures 10, 11, and 12 we could argue that the demand functions were largely derived demand (i.e., as discussed in Lecture 6 the market for most agricultural outputs is a derived demand based on the use of the commodity to create another product that is actually consumed – wheat is used to make bread).
- As such, we could simply use the concept that the derived demand curve is the marginal value product without any reference to utility or the consumption problem.
- In the next couple of lectures we return to the measurement of consumer surplus as a Willingness-To-Pay measure.

A Restatement of the Willingness-To-Pay



- In discussing Figure 4 it is important to recognize that it is a little different than most isoquants we have dealt with.
 - While the horizontal axis is q (a typical good), the vertical axis is y which we could interpret as the amount of money that we spent on all other goods.
 - The point a then gives the amount of q purchased as a function of the price of the good p_1 and the level of income m_1 .
 - Note that when we construct y basically as the amount of money spent on other goods we can graph its level on the same axes as we do income (m).
 - The budget constraint also is a little 'nonstandard'

$$p_1 q_1 + y = m_1 \quad (1)$$

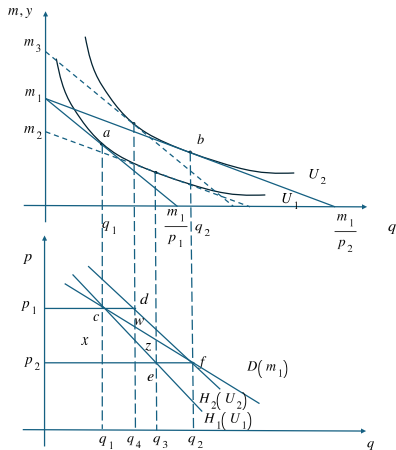
The implicit price of y is 1, and it will be 1 in the other budget lines.

- The tangency between the isoquant and the utility function at point a implies that y_1 in this 'dollar denominated good' and q_1 of the standard good.
- Next, we consider reducing the price of the ordinary good from p_1 to p_2 – the price line becomes flatter and the amount of q consumed increases.
 - The flattening of the budget line results in a new intersection point from m_1/p_1 to m_1/p_2 .
 - This yields an optimal point b with quantities y_2 and q_2 .
- To derive the *compensating variation* we consider constructing a new 'budget line' with the slope at the new price p_2 that is tangent with the original isoquant U_1 (i.e., at point c).
 - *Compensating Variation* – the amount of income that can be taken away which leaves the consumer indifferent with the original solution.
 - In this case, this amount is $m_1 - m_2$ (we will note as C_{12}).

- *Equivalent Variation*

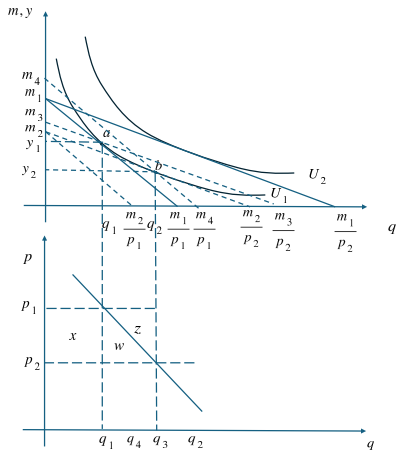
- Defined as amount of money that must be given to a consumer in lieu of the price and income changes.
- Here, this amount is based on point d – the Equivalent Variation (E_{12}) is defined as $m_3 - m_1$.
- Note that point d is defined by the tangency between new isoquant (U_2) and the budget constraint which is parallel to the old price line ($\overline{m_1 \frac{m_1}{p_1}}$).
- As suggested earlier, compensating and equivalent variation can be given more strict WTP interpretations.
 - For example, compensating variation for a price fall can be interpreted as the maximum amount the consumer would be willing *willing to pay* rather than relinquish the price reduction.
 - For a price increase, compensating variation is the minimum amount the consumer must pay, or is willing to accept, to tolerate the higher price.

- Equivalent variation, on the other hand, measures the gains and losses to the consumer associated with *not taking* the proposed action.
 - That is, equivalent variation for a price fall can be interpreted as the minimum amount the consumer is *willing to accept* to forgo the lower price.
- The *Hicksian compensated demand curve* is a relationship giving quantities demanded at various prices when utility is held constant by varying income.
- The *Marshallian demand curve* is a relationship giving the quantities demanded at various prices when income is held constant by varying utility.



- The Hicksian compensated demand curve going through point c in the lower panel of Figure 9 is generated from point a in the top panel.
 - The slope of the demand curve determines the level of q that will be purchased to maintain utility at U_1 .
 - Thus, as the price is reduced to p_2 and the compensating variation of $m_1 - m_2$ is taken away from the consumer to restore the original welfare level, the consumption increases to q_3 which implies a second point e on the compensated demand curve H_1 .
 - The increase in consumption from q_1 to q_3 , as a result of the price fall, is due entirely to the desire to substitute q for the other goods as the relative price of q falls.
- One can show that the compensating variation associated with the fall in price from p_1 to p_2 is exactly the area x and, hence, is equal to the vertical distance $m_1 - m_2$.

- Similarly, the compensating variation associated with an increase in price from p_2 to p_1 can be shown to be the negative of the area $w + x + z$ which is equal to $E_{12} = -(m_1 - m_3) = m_3 - m_1$. This area is also the equivalent variation for the price declining from p_1 to p_2 .
- In general, $C \leq \Delta S \leq E$ which differ by the amount of the areas z and w .
- Turning to Figure 12, we start by considering a decrease in the price of the good from p_1 (at point a) to p_2 . As this occurs, the consumer's utility increases from U_1 to U_2 .
- Conceptually, at the new price, an income of m_2 is required to generate the original level of utility – with a tangency at point b .
 - This movement implies a welfare gain of $m_1 - m_2 = x + w$ in the lower panel.
 - This is the compensating variation.



- If we increase the income from m_2 to m_3 , there income is sufficient to allow for the consumption of the original point at the new prices.
- Note that $m_1 = p_1 q_1 + y_1$ and $m_3 = q_2 q_1 + y_1$, therefore $m_1 - m_3 = (p_1 - p_2) q_1$ which is the inner cost difference of the original bundle of goods and is measure by the area x .
- Next, we consider the upper bound on the compensating variation. If we start at point b , the income is m_4 at a price of p_1 .
- The conjecture is that it would take a income payment of $m_4 - m_2$ to make the budget (y_2, q_2) feasible.
- $m_4 - m_2 = (p_1 - p_2) q_2 = w + x + z$ is the outer cost difference.