

An Introduction of Trade: Lecture XII

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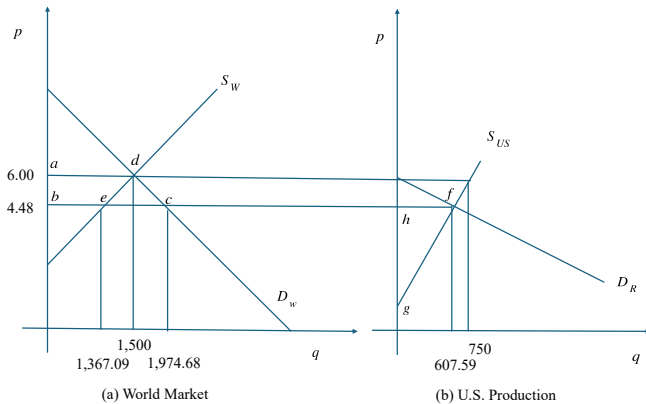
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- 1 Residual Demand Models
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Residual Demand Models

- One way to introduce trade is to contemplate what is known as a residual demand model.
- This model is useful for other reasons such as dominant firm arguments.
- In Figure 4 we will let Panel a be the world market for wheat:
 - We assume that the market price is \$ 6/bushel, the quantity produced and consumed at this price is 1,500 million bushels.
 - We assume that the elasticity of demand is -1.25 and the elasticity of supply is 0.35

$$\begin{aligned} D_W(p) &= \alpha_0^D + \alpha_1^D p = 3,375 - 312.5p \\ S_W(p) &= \alpha_0^S + \alpha_1^S p = 975 + 87.5p \end{aligned} \tag{1}$$



- First, we define the excess demand as the quantity demanded minus the quantity supplied

$$D_R(p) = \beta_0^R + \beta_1^R p = 2,400 - 400p. \quad (2)$$

- We have to construct a U.S. Supply function. I will do this by assuming that at \$ 6.00, the U.S. would produce 750 million bushels of wheat. In addition, I assume that the elasticity of domestic supply is 0.75

$$S_{US}(p) = \beta_0^{US} + \beta_1^{US} p = 187.5 + 93.75p. \quad (3)$$

- As a starting point, we assume that the domestic market for wheat is competitive and solve for the point where the residual demand equals the US supply

$$\Rightarrow 2,212.5 = 493.75p \Rightarrow p = 4.4810 \quad (4)$$

- To complete the model, we compute the amount of grain produced in the United States - 607.59.
- The amount of grain produced in the rest of the world (plugging the price into the world supply curve) 1,367.09.
- The amount of grain demanded by the rest of the world (plugging the price into the world demand curve) 1,974.69.
- Note that the difference is equal to the amount produced in the United States.

Rents

- The gain consumer surplus in the external market is $abcd$

$$\Delta S_W = (6.00 - 4.48) \times 1,500 + (6.00 - 4.48) \times (1,974.68 - 1,500) / 2 = 2,639 \quad (5)$$

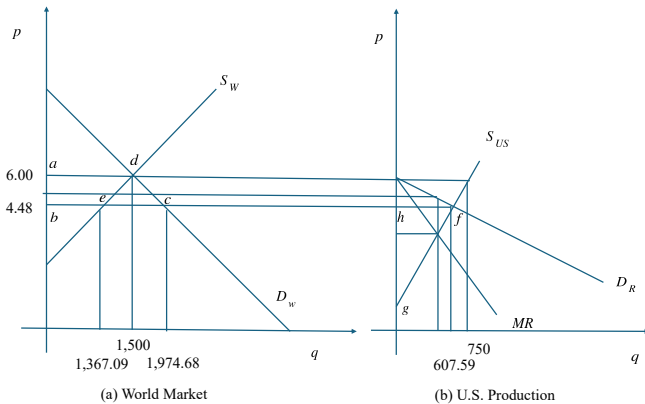
- The loss to international producers is $abed$

$$(6.00 - 4.48) \times 1,367.09 + (6.00 - 4.48) \times (1,500 - 1,369.09) / 2 = 2,177.54 \quad (6)$$

- Note that the area ecd represents the gains to trade.

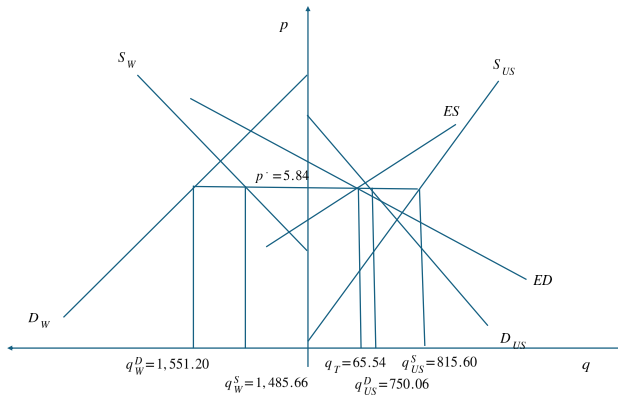
- Note that the intercept for the supply function is actually negative (as $q \rightarrow 0$, $q \rightarrow -\beta_0^S / \beta_1^S$).
- So, we put a choke point in at 1.00 yielding a producer welfare of

$$\Delta S_{US} = (4.48 - 1.00) \times 281.25 + (4.48 - 1.00) \times (606.59 - 281.25) / 2 = 1,547.04 \quad (7)$$



Back-to-Back Trade Diagram

- Figure 11 presents the Back-to-Back Trade Diagram.
- The goal is to do a more complete job representing the production/consumption relationships in the United States.
- As a starting point, we will keep the supply and demand relationships for the rest of the world developed in Equation 1.
- In a way similar to the residual demand model, subtracting the demand on the left-hand side of the model from the rest of the world's demand yields the *Excess Demand Curve* for the result of the world.
- Next, we want to derive a supply and demand function for wheat in the United States. As a starting point, we will use the most recent production data – 773 million bushels produced at a price of \$ 5.38 / bushel.



- Assuming an elasticity of demand of -0.35 and an elasticity of supply of 0.65

$$\begin{aligned} q_{US}^D &= 1,043.55 - 50.29p \\ q_{US}^S &= 270.55 + 93.39p \end{aligned} \quad (8)$$

- Subtracting the U.S. Demand curve from the U.S. Supply curve yields

$$ES(p) = -773 + 14.68p \quad (9)$$

- Solving for the equilibrium

$$\begin{aligned} [2,400 - 400p] - [-773 + 143.68p] &= \\ 3,173 - 543.68p = 0 &\Rightarrow p = 5.84 \end{aligned} \quad (10)$$

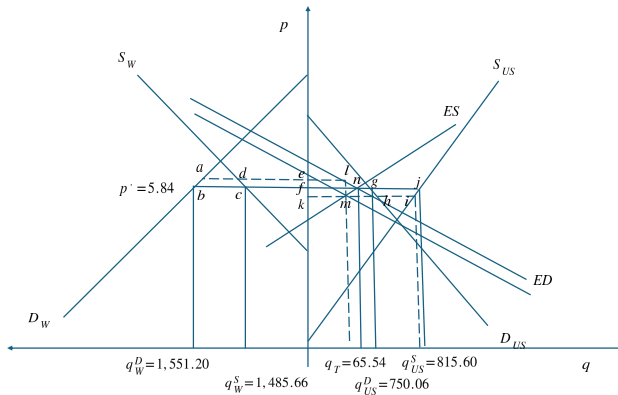
- So - what is the policy problem. Assume that the rest of the world places a \$ 0.10 bushel tariff on wheat imports.
- The Excess Demand curve shifts downward, as depicted in Figure 14

$$\begin{aligned} ED(p) &= 2,400 - 400(p + \tau) \\ &= 2,360 - 400p \end{aligned} \tag{11}$$

- Solving for the new equilibrium

$$\begin{aligned} [2,360 - 400p] - [3,173 - 543.68p] &= \\ 3,133 - 543.68p = 0 \Rightarrow p &= 5.7626 \end{aligned} \tag{12}$$

So the new U.S. price is 5.76 (lower than 5.83) and the new world price (after the tariff) is $5.76 + 0.10 = 5.86$ (higher than it was before).



- The quantity demanded in the rest of the world is 1,542.94 million bushels and the quantity supplied is 1,487.98.
- The quantity produced in the United States is 808.73 while the quantity demanded is 753.76.
- The quantity traded falls to 54.97 – the tariff collected is \$ 5.50 million.
- Working through the losses - first the net loss in the world market is *abcd*.
 - So consumers in the foreign markets lose *abfe*, but producers in the foreign market gain *dcfe*.

$$\begin{aligned}\Delta S_W = & (5.86 - 5.83) \times (1,551.20 - 1,542.94) / 2 + \\ & (5.86 - 5.83) \times (1,542.94 - 1,487.98) + \\ & (5.86 - 5.83) \times (1,487.98 - 1,485.66) / 2 = 1.59\end{aligned}\quad (13)$$

- Following similar logic, the United States loses $ghij$.
 - Again, producers in the United States lose $fki j$, but consumers in the United States gain $fkhg$

$$\begin{aligned}\Delta S_{US} = & (5.84 - 5.76) \times (753.76 - 750.06) / 2 + \\ & (5.84 - 5.76) \times (808.73 - 753.76) + \\ & + (5.84 - 5.76) \times (815.60 - 808.73) / 2 = 4.43\end{aligned}\tag{14}$$

- The DWL is lmn

$$\begin{aligned}DWL = & (5.86 - 5.84) \times (65.54 - 54.97) / 2 + \\ & (5.84 - 5.76) \times (65.54 - 54.97) / 2 = 0.51\end{aligned}\tag{15}$$