

Log-Log Functions and Wheat - An Application to Wheat: Lecture XI

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The Log-Log Function

- In the last lecture we used linear supply and demand curves.
- In this lecture, I would like to expand our applications slightly with the Log-Log functions.

$$\ln(q) = \alpha_0 + \alpha_1 \ln(p) \quad (1)$$

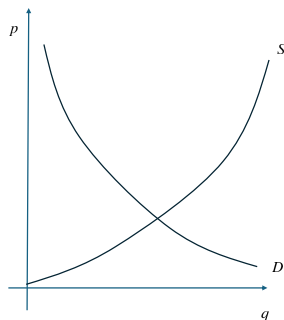
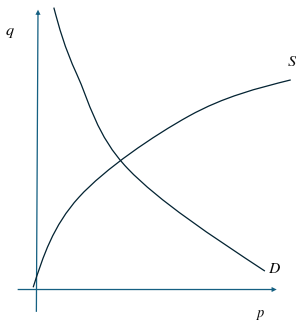
- 1 Note that this function has a constant elasticity

$$\begin{aligned} q = \exp(\alpha_0 + \alpha_1 \ln(p)) &\rightarrow \frac{\partial q}{\partial p} = \frac{\alpha_1 \exp(\alpha_0 + \alpha_1 \ln(p))}{p} \\ \Rightarrow \eta = \frac{\alpha_1 q}{p} \times \frac{p}{q} &= \alpha_1 \end{aligned} \quad (2)$$

- 2 The concavity can be derived as

$$\frac{\partial^2 q}{\partial p^2} = \alpha_1 \frac{p-1}{p^2} \exp(\alpha_0 + \alpha_1 \ln(p)) \quad (3)$$

- If we are modeling demand $\alpha_1 \ll 0$ which yields a downward sloping curve. In addition, as long as our initial price is greater than 1, the function is convex.
- Similarly, if $\alpha_1 \gg 0$ we have a representation of the supply function which is concave.
- Note that this concavity and convexity is in quantity as a function of price space, not the Marshallian price is equal to a function of quantity space.



Hard Red Winter Wheat

- Hard Red Winter Wheat is grown in much of the “Southern Great Plains” (eastern Colorado, Kansas, western Oklahoma, and the Texas panhandle).
- In the 2024/2025 crop year, 773 million bushels were produced at an average farm price of \$ 5.38 / bushel.
- Letting α_1 be the slope of the Log-Log demand curve and setting the elasticity of demand to be -0.45

$$\begin{aligned}\alpha_1 &= \ln(773) + 0.45 \ln(5.38) = 7.4075 \\ \Rightarrow q^D &= \exp(7.4075 - 0.45 \ln(p))\end{aligned}\quad (4)$$

- Similarly, if we assume that the elasticity of supply is 0.50

$$\begin{aligned}\beta_1 &= \ln(773) + 0.50 \ln(5.38) = 5.8089 \\ \Rightarrow q^S &= \exp(5.8089 + 0.50 \ln(p))\end{aligned}\quad (5)$$

- Check the equilibrium

$$\begin{aligned} 7.4075 - 0.45 \ln(p) &= 5.8089 + 0.50 \ln(p) \\ 1.5986 &= -0.95 \ln(p) \Rightarrow \ln(p) = 1.6827 \end{aligned} \tag{6}$$

which implies that $p = 5.38$.

Agricultural Policy Then and Now

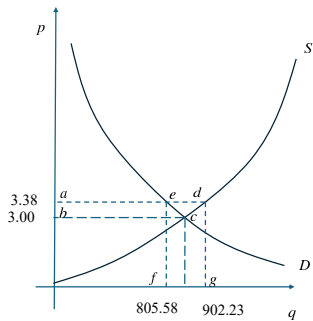
- In past lectures, we have discussed the allocation program. My home farm had a 50 acre cotton allotment.
- Another policy was the non-recourse loan program. Under this program, the government would purchase any quantity produced at a stated loan rate. This led to an accumulation of government stocks.
- A secondary program was called the deficiency payment program which paid the difference between the deficiency price and the maximum of the season average market price or the loan rate on historical production.
- Believe it or not each of these programs still exists in one way or another.

- The current loan rate is \$ 3.38/bu. for hard red winter wheat, and the reference price is \$4.00.
- The current market price is above payment, so consider a slightly different initial equilibrium at a market price for wheat of \$3.00 and a quantity supplied of 850 million bushels

$$\begin{aligned}\ln(q_D) &= 7.2396 - 0.45 \ln(p) \\ \ln(q_S) &= 6.1959 + 0.50 \ln(p)\end{aligned}\tag{7}$$

- First, consider the quantity supplied and demanded at \$ 3.38/bu.

$$\begin{aligned}\ln(q_S) &= 6.1959 + 0.50 \ln(3.38) \Rightarrow q_S = 902.23 \\ \ln(q_D) &= 7.2396 - 0.45 \ln(3.38) \Rightarrow q_D = 805.58\end{aligned}\tag{8}$$

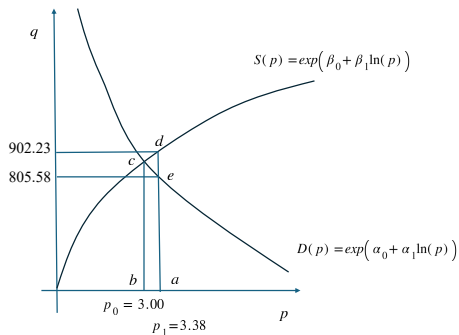


- Let us start with the way that the program was originally implemented. As in Equation 8, the original or market clearing solution is a price of \$ 3.00 and a quantity of 850 million bushels. At a loan rate of \$ 3.38 / bushel, producers produce 902.23 million bushels and consumers demand 805.58 million bushels.
- Under this program, the government had to buy 96.64 million bushels for the market to clear. So the government purchases $efgd = \$ 326.66$ million.
- Producers gain the area $abcd$ and consumers lose $abce$.
- As a starting point, consider how we can compute these surpluses.

- Using the linear approximation

$$\begin{aligned}\Delta S_P &= 850 \times (3.38 - 3.00) + \\ (902.23 - 850) \times (3.38 - 3.00) / 2 &= 332.92 \\ \Delta S_C &= 805.58 \times (3.38 - 3.00) + \\ (850 - 805.58) \times (3.38 - 3.00) / 2 &= 314.56\end{aligned}\tag{9}$$

- However, as an alternative consider the “rotated” graph presented in Figure 13



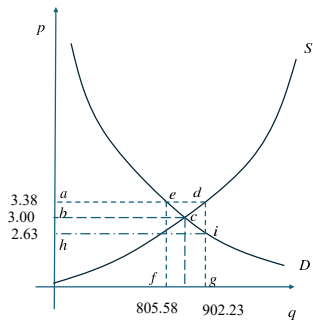
- Intuitively, we can return to an integral definition of the surpluses

$$\begin{aligned}\Delta S_P &= \int_{3.00}^{3.38} \exp(6.1959 + 0.50 \ln(p)) dp = 333.012 \\ \Delta S_C &= \int_{3.00}^{3.38} \exp(7.2396 - 0.45 \ln(p)) dp = 314.314\end{aligned}\tag{10}$$

- Note that the two solutions are very close - 332.924 versus 333.012 (0.000266) and 314.561 versus 314.314 (-0.00079).
- Basically the difference is the concavity and convexity of the functions over the range evaluated.

Loan Deficiency Payments

- The accumulation of stocks makes this program very undesirable; hence, the program was replaced with something called a Loan Deficiency Payment which is the difference between the loan rate and a computed market average price based on the price in the closest terminal markets and transportation cost.
- The argument is that this basically changes the program into a loan floor as depicted in Figure 16



- Computing the price that has to be charged for the market to clear

$$\ln(p) \frac{7.2396 - \ln(902.2287)}{0.45} \Rightarrow p = 2.6277 \quad (11)$$

- Treasury Cost $(3.38 - 2.6277) \times 902.23 = 678.7741$.
- Change in consumer surplus $bhic - 326.204$.
- Note that the change in producer surplus does not change – 332.92.
- Note that the DWL is cid which is 19.6463.

A Further Complication – Deficiency Payments

- The Agricultural and Consumer Protection Act of 1973 introduced the *deficiency payment* which was a 'quasi-fixed' payment which compensated farmers for prices lower than an 'equitable market price.'
- Specifically, the deficiency payment was the difference between a *target price* and the maximum of either an average annual market price or the loan rate times 80% of a five-year average quantity of production on recognized *program acres*.

$$DP = (p_t^{TP} - \hat{p}_t) \frac{0.80}{5} \sum_{i=1}^5 f(x_{t-i}) \quad (12)$$

where $\hat{p}_t$ is the greater of either the market price or the loan rate at time t .

- The deficiency payment shifts the supply curve a little bit because producer decisions effect the payment

$$\begin{aligned}\pi^* &= \hat{p}_t f(x_t) + \frac{0.80}{5} \sum_{i=1}^5 \delta^i (p_{t+i}^{TP} - \hat{p}_{t+i}) f(x_t) \\ \Rightarrow \frac{\partial \pi^*}{\partial x_t} &= \left[\hat{p}_t + \frac{0.80}{5} \sum_{i=1}^5 \delta^i (p_{t+i}^{TP} - \hat{p}_{t+i}) \right] \frac{\partial f(x_t)}{\partial x_t}\end{aligned}\quad (13)$$

Or slightly different in a profit function

$$\pi = \left[p_t + \frac{0.80}{5} \sum_{i=1}^5 \delta^i (p_{t+i}^{TP} - \hat{p}_{t+i}) \right] y - c(w, y). \quad (14)$$

- What does this mean?
 - It appears that farmers respond to both the current price and the target price.
 - However, we have to deal with uncertainty - the expectation that there will be deficiency payments over the next five years. Also, what about risk aversion (the certainty equivalent of an uncertain flow).
 - These payments were made on based or program acres.
- This general type of a payment has continued into the current farm bill as the *Price Loss Coverage* program. However, the program payments are now fixed to 85% of proven yields on base acres.

