

Multiple Demand Curves: Lecture IXb

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March 24, 2026

1 Looking Forward to the Hicksian System

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- Let us start by considering a total differentiation of the expenditure function

$$de(p, U) = e_{p_1}(p, U) dp_1 + e_{p_2}(p, U) dp_2 + \cdots e_{p_N}(p, U) dp_N$$

$$\int de(p, U) = \sum_{j=1}^N \int_L e_{p_j}(p, U) dp_j \quad (1)$$

where L represents some “path” of integration.

- So, we could rewrite Equation 1 as

$$\int de(p, U) dp = \sum_{j=1}^N \int_{p_{j0}}^{p_{j1}} e_{p_j}(\tilde{p}(p, U), U) dp_j \quad (2)$$

Nuts and Bolts

- First note that the term $\tilde{p}(p)$ refers to an ordering of the changes in prices – in a standard sum we start by changing p_1 in the integral holding the rest of the prices constant. In the second sum, we hold p_1 and $p_3 \cdots p_N$ constant and do the integral in p_2 space.
- Substituting for the Hicksian demand system

$$\int de(p, U) dp = \sum_{j=1}^N \int_{p_{j0}}^{p_{j1}} q_j^H(\tilde{p}(p), U) dp_j \quad (3)$$

- Note that similar to the Hicksian demand system

$$d\pi(p, w) = d(p'q - c(w, y)) \quad (4)$$

- So by extensions

$$S = \sum_{j=1}^N \int_{w_{0j}}^{w_{1j}} x_j(\tilde{w}(w), y) dw_j \quad (5)$$

- As a complicating factor, I would like to introduce the Social Welfare Function that we use with a Takayama and Judge spatial formulation

$$W = \int_{q_{10}}^{q_{11}} \int_{q_{20}}^{q_{21}} \left(\alpha' q + \frac{1}{2} q' A q \right) dq_2 dq_1 \quad (6)$$

Note first that the Takayama and Judge formulation is specified using “inverse demand functions” – the price that results from a given quantity. However, note that applying Leibniz’s rule, differentiating Equation 6 yields the Surplus formulation that we have been working with.